PROGRESSIVE SCHOOL SERIES.

ACADEMIC ARITHMETIC,

BEING PART FOUR OF THE

PROGRESSIVE SCHOOL SERIES OF ARITHMETICS.

BY

W. T. KENNEDY,

Principal of County Academy, Halifax,

AND

PETER O'HEARN,

Principal of St. Patrick's Boys' High School, Halifax.

T. C. ALLEN & CO.,

HALIFAX, NOVA SCOTIA.

PREFACE.

This part, No. IV, is intended, with the parts already published, to complete our Arithmetic.

The parts I, II, and III, previously issued, cover the Common School Course up to and including Grade VIII. Part IV has, besides new subjects, exercises of a more difficult character than those of I, II, and III, and is designed for use in Academic and High Schools. The four parts are so graded and correlated that they make a complete, continuous text book on Arithmetic for Primary and Secondary Schools without overlapping each other, an advantage to teachers, pupils and parents which will be apparent.

The problems, we believe, will be found to be thoroughly practical. Puzzling questions, long and difficult exercises and subjects that do not afford a mental training have been avoided. We think that a knowledge of principles can be more readily acquired by working many comparatively easy exercises than by the use of those of a perplexing nature.

It is hoped that the treatment of the subjects throughout this part, and especially those not usually found in school arithmetics, will commend itself to teachers.

W. T. KENNEDY.
P. O'HEARN.

CONTENTS.

Exa

Simp

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.

11.

12.

13.

14.

COMPLEX FRACTIONS.

EXAMPLE 1. Simplify $3\frac{1}{12} - 5\frac{1}{2} \div 4\frac{1}{3} \times 1\frac{2}{11} - \frac{5}{11}$ of $(\frac{5}{6} - \frac{4\frac{1}{2}}{12})$.

$$3\tfrac{1}{12} - 5\tfrac{1}{2} \div 4\tfrac{1}{3} \times 1\tfrac{2}{11} - \tfrac{5}{11} \text{ of } (\tfrac{5}{6} - \tfrac{4\frac{1}{2}}{12})$$
$$= 3\tfrac{1}{12} - \tfrac{11}{2} \div \tfrac{13}{3} \times \tfrac{13}{11} - \tfrac{5}{11} \text{ of } (\tfrac{5}{6} - \tfrac{3}{8})$$
$$= 3\tfrac{1}{12} - \tfrac{11}{2} \times \tfrac{3}{13} \times \tfrac{13}{11} - \tfrac{5}{11} \text{ of } \tfrac{11}{24}$$
$$= 3\tfrac{1}{12} - 1\tfrac{1}{2} - \tfrac{5}{24} = 1\tfrac{7}{12} - \tfrac{5}{24} = 1\tfrac{9}{24} = 1\tfrac{3}{8}$$

EXERCISE 1.

Simplify :

1. $\frac{2}{3}$ of $\frac{3}{4} \div \frac{5}{6}$ of $(\frac{7}{10} - \frac{2}{3}$ of $\frac{9}{10})$.
2. $5\frac{1}{2} + 3\frac{1}{3} \times 4\frac{2}{10} - 4 \times 2\frac{1}{7}$.
3. $16\frac{2}{7} + 10\frac{3}{14} - 1\frac{1}{5} - 4\frac{1}{5}$ of $\frac{3}{49}$.
4. $3\frac{1}{3}$ of $5\frac{1}{5} - \frac{3}{4} \div \frac{7}{8} + 1\frac{1}{3} \div 3\frac{1}{9}$.
5. $\frac{1}{4\frac{5}{9}}$ of $8 - 2\frac{1}{2}$ of $\frac{1}{15} - \frac{1}{9} \div \frac{7}{3}$.
6. $3\frac{3}{7} \div \frac{7}{32}$ of $5\frac{5}{7} - 1\frac{2}{7}$.
7. $5\frac{1}{8}$ of $\frac{7}{8} + \frac{2}{3}$ of $\frac{5}{12} \div 6$.
8. $24 - 1\frac{1}{8} \div 3\frac{3}{5} + 4\frac{5}{9} \div 4\frac{1}{10}$.
9. $\frac{10\frac{1}{2}}{7\frac{1}{3}}$ of $8\frac{8}{9} - \frac{5}{11} + 1\frac{5}{6}$ of $\frac{3}{121}$
10. $\frac{3}{7}$ of $\dfrac{6\frac{3}{4} - \frac{2}{3}}{2 - 1\frac{1}{4}} \div \frac{2}{3}$ of $\frac{9}{11}$.
11. $18 - 20\frac{1}{4} \times 3\frac{1}{9}$ of $\frac{1}{28} - \frac{3}{16}$.
12. $1\frac{1}{8}$ of $\dfrac{16\frac{1}{2} - 4\frac{5}{6}}{7\frac{2}{3} + 6\frac{3}{16}}$ of $\dfrac{6\frac{10}{11}}{4\frac{1}{5}$ of $9\frac{1}{11}}$.
13. $\dfrac{2\frac{1}{7} \times \frac{1}{3} + \frac{4}{5}$ of $3\frac{3}{4}}{3\frac{1}{9} \times \frac{6}{7} - 2\frac{1}{5} \div 1\frac{4}{7}}$.
14. $10\frac{5}{19} - (2\frac{3}{4} + 7$ of $\frac{25}{38} - \dfrac{1\frac{2}{3}}{2\frac{1}{2}}) \div (2 - \frac{151}{228})$.

15. $2\frac{1}{3}$ of $\dfrac{5\frac{1}{4}}{1\frac{1}{3}} - 4\frac{1}{8} \times 3\frac{3}{11} \div \dfrac{3\frac{3}{7}}{2\frac{1}{4}}$.

16. $1 - \left(\frac{1}{7} \times \dfrac{1}{\frac{3}{4}\frac{7}{9}} - \frac{2}{13} \text{ of } 1\frac{2}{3\frac{7}{}} + \dfrac{\frac{1}{3}}{\frac{3\frac{7}{}}{2\frac{4}{}}}\right)$.

17. $3 \div \left(\frac{1}{16} \div \dfrac{1}{7\frac{3}{5}} + \frac{3}{7} \text{ of } 10\frac{1}{5} - \frac{5}{7} \text{ of } 2\frac{9}{10}\right)$.

18. $37 - \left(3\frac{1}{4} \div \dfrac{\frac{1}{2} - \frac{1}{13}}{1\frac{1}{2} + 3\frac{1}{4}}\right)$.

19. $(4 - 3\frac{2}{3}) \times (\frac{1}{2} \text{ of } 2\frac{1}{4} \div 4\frac{1}{2}) \times 1\frac{2}{13}$.

20. $\dfrac{(\frac{1}{2} + \frac{1}{3}) \times (\frac{1}{4} + \frac{1}{5}) - (\frac{1}{2} - \frac{1}{3}) \times (\frac{1}{4} - \frac{1}{5})}{(\frac{1}{2} + \frac{1}{3}) \times (\frac{1}{4} - \frac{1}{5}) \div (\frac{1}{6} - \frac{1}{7})}$.

21. $\dfrac{\frac{5}{6} \text{ of } \frac{6}{7} - \frac{3}{14} \text{ of } 2}{\frac{7}{14}\frac{8}{4} \times \frac{2}{4}\frac{1}{2}\frac{8}{5} \div \frac{3}{2}\frac{2}{8}\frac{7}{9} \times \frac{7}{1}\frac{5}{1}\frac{}{7}}$.

22. $13\frac{4}{9} - 2\frac{1}{10} \text{ of } \frac{11}{14} \text{ of } \dfrac{1}{4\frac{2}{5}} + 4\frac{1}{5} \text{ of } 3\frac{4}{7} - \dfrac{7\frac{1}{9}}{3\frac{1}{5}}$.

23. $11\frac{1}{9} - \left(\dfrac{1}{5\frac{1}{4}} + \dfrac{3\frac{1}{3}}{7} - \dfrac{15}{21} \times \dfrac{4}{18}\right) + 3 \div \frac{2}{3} \text{ of } \frac{3}{4}$.

24. $\frac{3}{3}\frac{9}{2} \text{ of } \frac{8}{30} - 3\frac{1}{3} \div 13\frac{1}{3} \text{ of } 1\frac{1}{19} + \frac{6}{7} \text{ of} \dfrac{6\frac{5}{12}}{1\frac{5}{6}}$.

25. $\dfrac{\frac{3}{5} \text{ of } 1\frac{2}{3} \text{ of } 21}{46} \text{ of} \dfrac{7\frac{2}{3}}{5\frac{1}{4}} \times \dfrac{2\frac{7}{8} \text{ of } \frac{3}{8} \text{ of } 9\frac{3}{5} \text{ of } 3\frac{11}{12}}{117\frac{13}{40} - 106\frac{14}{15}} \times 1\frac{1}{2}$.

Write in one expression and simplify the following :

26. The difference between the sum and difference of $6\frac{3}{4}$ and $\frac{2}{3}$, multiplied by their product.

27. The sum of $9\frac{3}{8}$ and $17\frac{5}{7}$ multiplied by the product of $1\frac{1}{55}$ and $2\frac{1}{2}$.

28. The difference of $18\frac{3}{4}$ and $1\frac{3}{8}$ increased by their sum, and the result divided by their product.

29. Divide the sum of $\dfrac{\frac{2}{3} \text{ of } \frac{3}{4}}{\frac{1}{2}}$ and $\dfrac{\frac{2}{5} \text{ of } \frac{5}{6}}{\frac{2}{9} \text{ of } 4\frac{1}{2}}$ by their difference.

30. What number multiplied by $\frac{2}{9}$ of $\frac{5}{8} \times 3\frac{2}{7}$ will produce $\frac{23}{84}$?

COMPLEX FRACTIONS.—Continued.

Any simple fraction in its lowest terms, whose numerator is greater than unity, can be put in the form of a complex fraction by dividing both terms by the numerator.

$\frac{15}{41} = \frac{1}{2+\frac{11}{15}}$. Reducing $\frac{11}{15}$ in the same manner and continuing the process as the simple fractions occur, we have

$$\frac{15}{41} = \frac{1}{2+\frac{11}{15}} = \frac{1}{2+\frac{1}{1+\frac{4}{11}}} = \frac{1}{2+\frac{1}{1+\frac{1}{2+\frac{3}{4}}}} = \frac{1}{2+\frac{1}{1+\frac{1}{2+\frac{1}{1+\frac{1}{3}}}}}$$

The numerator of the last fraction will always be 1.

Such fractions are called **Continued Fractions,** and they can be simplified by reversing the above operation.

$$\frac{1}{2+\frac{1}{1+\frac{1}{2+\frac{1}{1+\frac{1}{3}}}}} = \frac{1}{2+\frac{1}{1+\frac{1}{2+\frac{1}{1\frac{1}{3}}}}} = \frac{1}{2+\frac{1}{1+\frac{1}{2+\frac{3}{4}}}} = \frac{1}{2+\frac{1}{1+\frac{1}{2\frac{3}{4}}}}$$

$$= \frac{1}{2+\frac{1}{1+\frac{4}{11}}} = \frac{1}{2+\frac{1}{1\frac{4}{11}}} = \frac{1}{2+\frac{11}{15}} = \frac{1}{2\frac{11}{15}} = \frac{15}{41}$$

EXERCISE 2.

Reduce to continued fractions and verify by reversing the operation as above :—

1. $\frac{11}{23}$, $\frac{13}{54}$, $\frac{15}{61}$.
2. $\frac{17}{40}$, $\frac{19}{36}$, $\frac{29}{40}$.
3. $\frac{10}{39}$, $\frac{109}{339}$, $\frac{29}{121}$.
4. $17\frac{50}{61}$, $20\frac{288}{1793}$, $11\frac{38}{163}$

EXERCISE 3.

1. Simplify $(6\frac{3}{8} - 4.25) \div (.005 \div 1.25)$.

2. Divide $2\frac{3}{4} \times 5.75$ by $\frac{3}{7}$ of 1.267.

3. Reduce $\frac{16}{171}$ to a decimal of the sixth place.

4. Simplify $\dfrac{.05 \times 27.3782}{43.267} \div \dfrac{13.6891}{865.34}$.

5. Find the quotient of $1 \div 1.007633$ to the sixth place.

6. Simplify $(6.35 - 4.5)\ (.6 - .005) \div (.0006 \times .005)$.

7. Simplify $\frac{.095 \div .0005}{.0087 \div .3} \div \frac{3.42 \div .006}{.000812 \div .04}$

8. Find the value of $\sqrt{(\frac{1}{7} + \frac{1}{14} + 10\frac{1}{21})}$ to 4 decimal places.

9. Simplify (1 mi. 7 fur. 39 rd. 5 yd. 1 ft. 7 in.) $\times$ 18.75.

10. Find the value of .06 of £10 + .10 of 6s. + .7 of 10d.

11. How many feet is the side of a square field containing .75 of an acre?

12. Express 4s. $5\frac{1}{4}$d. as the decimal of $\frac{7}{20}$ of 7s. $4\frac{3}{4}$d.

13. Express in yards 7.16 miles – 3.1407 furlongs.

14. Express in square inches .07 of an acre – .07 of a square rod.

15. Find the value of $\dfrac{1.5}{.075} \times \dfrac{3.25}{1.5} \times \dfrac{1}{86\frac{2}{3}}$.

RECURRING DECIMALS.

In reducing some fractions to decimals the division will not be exact, there being a remainder no matter how far the operation is carried.

Thus, $\frac{3}{8} = .375$, the division terminating when the third order is reached.

$\frac{1}{3} = .333+$, the division never terminating.

A **Finite Decimal** is one in which the number of decimal places is limited.

An **Infinite Decimal** is one in which the number of decimal places is unlimited.

A **Recurring Decimal** is an infinite decimal in which one or more figures recur in the same order; as, $.666+$; $.252525+$; $.6389389389+$.

The figure, or set of figures, that is repeated is called a **Repetend.** In the above, 6 is the repetend of the first decimal, 25 the repetend of the second, and 389 the repetend of the third.

When a repetend consists of one digit it is indicated by a point placed over it; when the repetend consists of more than one digit a point is placed over the first and last figures.

Thus:—$.666+$, $.252525+$, $.6389389389+$ are written $.\dot{6}$, $.\dot{2}\dot{5}$, $.6\dot{3}8\dot{9}$.

NOTE.—Recurring decimals are also called *circulating* or *repeating* decimals.

A Pure Recurring Decimal contains no figures but those of the repetend; as, $.\dot{6}$, $.\dot{2}\dot{5}$, $.\dot{4}3\dot{5}$.

A Mixed Recurring Decimal contains one or more figures (called the finite part) written between the decimal point and the repetend; as, $.4\dot{5}$, $.643\dot{4}\dot{5}$.

Any number containing no other factors than 2 or 5 is itself a factor of some power of 10, because 2 and 5 are factors of 10. If a number contains any other factor it cannot be itself a factor of any power of 10, because 2 and 5 are the only factors of 10.

When a fraction is in its lowest terms it can be reduced to a finite decimal only when its denominator contains no other factors than 2 and 5. It can then be reduced to an equivalent fraction having

some power of 10 for its denominator, and consequently to a finite decimal.

$$\text{Thus } \frac{5}{8} = \frac{5}{2\times2\times2}, \text{ and } \frac{5}{8} = \frac{5\times125}{8\times125} = \frac{625}{1000} = .625.$$

$$\text{Also } \frac{9}{40} = \frac{9}{2\times2\times2\times5} \text{ and } \frac{9}{40} = \frac{9\times25}{40\times25} = \frac{225}{1000} = .225.$$

NOTE. When the denominator of a fraction has any other prime factor than 2 or 5, it is evident the fraction cannot be reduced to a finite decimal.

When a fraction in its lowest terms is reduced to a finite decimal the number of decimal places will be equal to the greatest number of equal factors, 2 or 5, in the denominator.

Thus in the fraction $\frac{9}{40} = \frac{9}{2\times2\times2\times5}$ the factor 2 occurs 3 times and the number of decimal places in the equivalent decimal, .225, is 3.

Every infinite decimal, derived from a common fraction, is also a recurring decimal; and the number of places in the repetend must be less than the number of units in the denominator of the common fraction. In the reduction of any fraction to a decimal the number of possible remainders is one less than the number of units in the divisor (denominator); thus, when 7 is a denominator the only possible remainders are 1, 2, 3, 4, 5, and 6. Hence some one of the remainders must repeat before the number of decimal places equals 7.

EXERCISE 4.

Express as recurring decimals :—

1. $\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{6}$.
2. $\frac{5}{7}$, $\frac{9}{14}$, $\frac{7}{22}$.
3. $\frac{1}{70}$, $\frac{7}{48}$, $\frac{1}{9}$.
4. $3\frac{1}{99}$, $200\frac{1}{81}$, $\frac{4\frac{3}{8}}{7\frac{6}{7}}$.

To express a Pure Recurring Decimal as a common fraction.

EXAMPLE 2. Express $.\dot{6}$ as a common fraction.

$.\dot{6} = .666\ldots$
Ten times the decimal $. = 6.66\ldots$
Once the decimal $= .66\ldots$
9 times the decimal $= 6$
The decimal itself $= \frac{6}{9} =$

EXAMPLE 3. Express $.\dot{2}0\dot{8}$ as a common fraction.

$.\dot{2}0\dot{8}$	$= .208208208\ldots$
1000 times the decimal	$= 208.208208\ldots$
Once the decimal	$= \quad .208208\ldots$
999 times the decimal	$= 208$
The decimal itself	$= \frac{208}{999}$.

To express a pure recurring decimal as a common fraction, write down the repetend for a numerator and as many nines as there are figures in the repetend for a denominator.

Any noughts occurring in the repetend on the left of the first significant digit are of course omitted; thus $.\dot{0}0\dot{1}8 = \frac{18}{9999}$.

EXERCISE 5.

Reduce to common fractions in their lowest terms :—

1. $.\dot{8}$, $.\ddot{4}\dot{5}$, $.\ddot{5}\dot{4}$.
2. $.2\dot{3}\dot{6}$, $.\dot{0}0\dot{5}\dot{4}$, $.\dot{0}00381\dot{}$.
3. $6.5\dot{4}\dot{4}\dot{5}$, $14.\dot{0}1\dot{6}$, $100.0\dot{5}\dot{9}$.
4. $1.\dot{0}0\dot{3}\dot{8}$, $18.\ddot{0}\dot{1}$, $60.\dot{0}00\dot{3}$.
5. $6.\dot{0}\dot{8}\dot{7}$, $.\dot{0}0\dot{4}\dot{0}$, $.\dot{0}1\dot{7}\dot{6}$.
6. $8.\dot{0}0\dot{8}$, $.\dot{1}00\dot{7}\dot{6}\dot{4}$, $.\dot{9}\dot{9}\dot{9}$.

To express a Mixed Recurring Decimal as a common fraction.

EXAMPLE. 4. Express $.65\ddot{4}$ as a common fraction.

The decimal	$= .65454\ldots$
1000 times the decimal	$= 654.54\ldots$
10 times the decimal	$= \quad 6.54\ldots$
990 times the decimal	$= 648$
The decimal itself	$= \frac{648}{990}$ or $\frac{654-6}{990}$

EXAMPLE 5. Express $.4\dot{2}4\dot{5}$ as a common fraction.

The decimal	$= .424545\ldots$
10,000 times the decimal	$= 4245.45\ldots$
100 times the decimal	$= 42.45\ldots$
9900 times the decimal	$= 4203$
The decimal itself	$= \frac{4203}{9900}$ or $\frac{4245-42}{9900}$.

To express a Mixed Recurring Decimal as a common fraction, subtract the finite part from the whole decimal and set down the remainder for a numerator; for a denominator write down as many nines as there are figures in the repetend, followed by as many noughts as there are figures in the finite part.

Thus: $.3\dot{4}\dot{7}\dot{5} = \frac{3475-34}{9900} = \frac{3441}{9900}$; $.07\dot{4}5\dot{6} = \frac{7456-7}{99900} = \frac{7449}{99900}$

EXERCISE 6.

Reduce to common fractions in their lowest terms:—

1. $.4\dot{5}$, $.5\dot{7}$, $.04\dot{8}$.
2. $.04\dot{8}$, $.6\dot{3}7\dot{6}$, $.6\dot{3}7\dot{6}$.
3. $.6\dot{3}7\dot{6}$, $.00\dot{7}\dot{9}$, $.0\dot{0}7\dot{9}$.
4. $.\dot{0}00\dot{8}$, $.00\dot{8}$, $.000\dot{8}$.
5. $6.2342\dot{3}5$, $17.0179\dot{3}$, $64.0\dot{1}$.
6. $4.000\dot{1}$, $4.000\dot{1}$, $29.0873\dot{1}$.

Similar Repetends are those which begin at the same decimal place.

Conterminous Repetends are those that end at the same decimal place.

$.40\dot{6}$, $.0\dot{7}\dot{3}\dot{8}$ and $.1\dot{6}7\dot{3}4$ are similar repetends, each repetend beginning at the third place.

$.4\dot{3}7\dot{8}\dot{9}$, $.0\dot{1}67\dot{5}$ and $.3\dot{6}591$ are conterminous, each ending at the fifth place.

PROPERTIES OF REPETENDS.

1. Any finite decimal may be regarded as a mixed recurring decimal by annexing noughts. Thus: $.46 = .46\dot{0}\dot{0} = .46000\dot{0}$, &c.

2. A repetend having any number of places may be changed into one having any number of times that number of places, without any change of value. Thus: $.6\dot{3}\dot{9} = .63939\dot{3}\dot{9} = .6393939\dot{3}\dot{9}$, &c.

3. Any repetend may be changed in form into another having a finite part and an infinite part without changing its value, the latter having as many places as the original repetend. Thus: $.5\dot{6}7\dot{8} = .56786\dot{7}\dot{8} = .567867\dot{8}\dot{7} = .5678678\dot{7}\dot{8}$.

Hence all recurring decimals can be made similar.

4. When two or more repetends are similar they may be made conterminous by extending the repetends as many places as the least common multiple of the places in each repetend.

EXAMPLE 6. Change $.\dot{6}$, $.0\ddot{4}\ddot{5}$ and $.36\dot{7}8\dot{9}4\dot{}$ to similar and conterminous repetends.

Dissimilar.		Similar.			Similar and Conterminous.	
$.\dot{6}$	=	$.666$	$\dot{6}$	=	$.666$	$\dot{6}66666\dot{}$
$.0\ddot{4}\ddot{5}$	=	$.045$	$\ddot{4}\ddot{5}$	=	$.045$	$\dot{4}54545\dot{}$
$.36\dot{7}8\dot{9}4\dot{}$	=	$.367$	$8\dot{9}4\dot{}$	=	$.367$	$\dot{8}94894\dot{}$

Here when the repetends are made similar they begin *after* the third place. The places in the different repetends are 1, 2 and 3. The L. C. M. of 1, 2 and 3 being 6, we extend the figures of each repetend six places from the third place.

EXAMPLE 7. Make .7, $.0\dot{4}\dot{3}$, $.6\dot{7}3\dot{8}5$ and $.2\dot{4}6\dot{7}8\dot{5}4$ similar and conterminous.

Dissimilar.		Similar.		Similar and Conterminous.
.7	=	$.7000\dot{}$	=	$.7000000000000000\dot{}$
$.0\dot{4}\dot{3}$	=	$.04343\dot{}$	=	$.0434343434343\dot{}$
$.6\dot{7}3\dot{8}5$	=	$.673853\dot{}$	=	$.6738538538538\dot{}$
$.2\dot{4}6\dot{7}8\dot{5}4$	=	$.2467854\dot{}$	=	$.246785478547854\dot{}$

In the above we change to similar repetends by (1) and (3), and then make the latter conterminous by (2) and (4). It is evident that if the figures were continued one place further the last column would be exactly the same as the fourth column, and so on.

ADDITION OF RECURRING DECIMALS.

EXAMPLE 8. Add $17.2\dot{3}$, $41.2\dot{4}76$, $8.6\dot{1}$, 1.5 and $35.4\dot{2}3$.

Dissimilar.	Similar.		Conterminous.	
$17.2\dot{3}$	= 17.2	$\dot{3}\dot{2}$	= 17.2	$3232\dot{3}\dot{2}$
$41.2\dot{4}76$	= 41.2	$\dot{4}76$	= 41.2	$476476\dot{}$
$8.6\dot{1}$	= 8.6	$1\dot{6}$	= 8.6	$161616\dot{}$
1.5	= 1.5	$0\dot{0}$	= 1.5	$000000\dot{}$
$35.4\dot{2}3$	= 35.4	$2\dot{3}$	= 35.4	$232323\dot{}$
				$104.0193648\dot{}$

In the above we make the decimals similar and conterminous and add. We carry, to be added to the right hand column, the number that would be carried if the decimals were continued two or more places further. This is the same number that is carried from the addition of the column where the similar repetends *begin*. In the above it is 1 and is carried from the sixth column, counting from the right, and added to the sum of the right hand column.

TO ADD RECURRING DECIMALS.

1. Make the repetends similar and conterminous.

2. Add as in finite decimals, taking care to increase the sum of the right hand column by as many units as are carried from the left hand column of the repetends.

3. Place the points in the result directly under the points above.

NOTE. Sometimes the results of operations in recurring decimals may be changed into equivalents containing fewer figures:

Thus, $.369875469\dot{=}.3698754\dot{}$.

EXERCISE 7.

Find the sum of:—

1. $2.\dot{4}$, $.3\dot{2}$, $.5\dot{6}7$, $7.05\dot{6}$ and $4.3\dot{7}$.
2. $.47\dot{8}$, $.3\dot{2}1$, $.7856\dot{4}$, $.3\dot{2}$, $.\dot{5}$ and $.432\dot{6}$.
3. $.\dot{9}$, $6.3\dot{2}\dot{7}$, $19.4\dot{3}$, $27.02\dot{7}\dot{8}$ and $.034712\dot{3}$.
4. $7.4\dot{2}\dot{7}$, $9.1\dot{2}3\dot{4}$, $17.2987\dot{6}4\dot{3}$ and $18.\dot{6}\dot{7}$.
5. $4.\dot{9}\dot{5}$, $7.1\dot{6}\dot{4}$, $4.71\dot{2}\dot{3}$ and $.9731\dot{7}$.
6. $97.\dot{2}\dot{6}$, $3.\dot{7}6923\dot{0}$, $2.\dot{9}\dot{3}$, $.8\dot{1}\dot{4}$, $.1\dot{6}\dot{2}$, 99.083, 1.5 and 134.09.

SUBTRACTION OF RECURRING DECIMALS.

EXAMPLE 9. From $62.\dot{8}\dot{6}$ take $18.\dot{3}7\dot{2}3\dot{5}$.

Similar and Conterminous.

$62.\dot{8}\dot{6}$	=	62.8	$\dot{6}8\dot{6}\dot{8}$
$18.\dot{3}7\dot{2}3\dot{5}$	=	18.3	$\dot{7}2\dot{3}\dot{5}$
		44.4	$\dot{9}6\dot{3}\dot{2}$

TO SUBTRACT RECURRING DECIMALS:

1. Make the repetends similar and conterminous and subtract as in whole numbers, putting the decimal point in its proper place and the

dots indicating the repetends directly under those in the subtrahend and minuend.

2. When a repetend of the subtrahend is larger than that of the minuend, 1 is to be added to the right hand figure of the subtrahend before subtracting.

In the above example the repetend of the subtrahend ($\dot{7}23\dot{5}$) is larger than ($\dot{6}86\dot{8}$) the repetend of the minuend. If the repetends were carried out one or more places it is evident that there would be a remainder of 2 instead of 3 in the fifth place.

EXERCISE 8.

1.	From	$75.\dot{6}\dot{2}$	take	$3.7643\dot{2}.$
2.	"	$476.3\dot{2}$	"	$184.769\dot{7}.$
3.	"	$13.856\dot{4}$	"	$.038\dot{2}.$
4.	"	$456.\dot{1}2\dot{3}$	"	$411.\dot{3}.$
5.	"	$901.\dot{6}$	"	$400.75.$
6.	"	$4.782\dot{4}$	"	$.\dot{8}\dot{7}.$
7.	"	2419.6	"	$1200.\dot{9}.$
8.	"	$.634\dot{8}5\dot{2}$	"	$.0\dot{2}\dot{1}.$
9.	"	$9482.\dot{4}2\dot{1}$	"	$6031.\dot{0}3\dot{5}.$
10.	"	1	"	$.\dot{9}$

MULTIPLICATION OF RECURRING DECIMALS.

EXAMPLE 10. Multiply $.\dot{3}\dot{6}$ by $.2\dot{5}$.

$$.\dot{3}\dot{6} = \frac{36}{99} = \frac{4}{11}.$$

$$.2\dot{5} = \frac{23}{90}.$$

$$\frac{4}{11} \times \frac{23}{90} = \frac{92}{990} = .0\dot{9}\dot{2}.$$

TO MULTIPLY RECURRING DECIMALS.

Reduce the given decimals to common fractions and multiply as usual; then reduce the product to a decimal.

EXERCISE 9.

Find the product of:—

1. $.\dot{4}\dot{5} \times .0\dot{6}$
2. $37.2\dot{3} \times .2\dot{6}$
3. $.1\dot{2}\dot{3} \times .\dot{6}$
4. $.2\dot{4}\dot{5} \times 7.\dot{3}$
5. $24.\dot{6} \times 15.\dot{7}$
6. $48.\dot{2}\dot{3} \times 16.1\dot{3}$
7. $8574.\dot{3} \times 87.\dot{5}$
8. $3.\dot{9}7\dot{3} \times 8$
9. $49640.54 \times .7050\dot{3}$
10. $7.\dot{7}\dot{2} \times .\dot{2}9\dot{7}$

DIVISION OF RECURRING DECIMALS.

EXAMPLE 11. Divide $.3\dot{5}$ by $.\dot{4}\dot{5}$.

$$.3\dot{5} = \frac{32}{90} = \frac{16}{45}$$

$$.\dot{4}\dot{5} = \frac{45}{99} = \frac{5}{11}$$

$$\frac{16}{45} \div \frac{5}{11} = \frac{176}{225} = .78\dot{2}.$$

TO DIVIDE RECURRING DECIMALS.

Reduce the divisor and dividend to common fractions; divide one by the other and change the result to a decimal.

EXERCISE 10.

1. Divide $.8\dot{4}$ by $.6\dot{4}$
2. " $18.\dot{5}\dot{6}$ by $.\dot{3}$
3. " $.6$ by $.1\dot{2}\dot{3}$
4. " $2.29\dot{7}$ by $.2\dot{9}\dot{7}$
5. " 2 by $.\dot{9}$
6. Divide $.\dot{9}$ by $.9$
7. " $630.10\dot{1}$ by $.\dot{3}\dot{3}$
8. " $.000\dot{3}\dot{6} \div 2.\dot{7}\dot{2}$
9. " $.\dot{6} \div 0\dot{6} \times .00\dot{6}$
10. " $.0\dot{3}\dot{6} \times \dot{9} \div \dot{3}$

NOTE.—It may be observed here that when the multiplicand is a recurring decimal and the *multiplier* is a *finite* number or decimal,

exact multiplication can be readily performed without reduction to a common fraction.

The repetend in the product in such a case will have the same number of places as the repetend of the multiplicand.

Also, when the dividend is a recurring decimal and the divisor a finite number or decimal, the operation of division can be easily performed without reducing the recurring decimal to a common fraction.

CUBE ROOT.

The **Cube,** or third power of a number, is the product of three factors, each of which is the number.

The cube of 4 is $4 \times 4 \times 4$, that is 64.

The cube is denoted by a small 3 placed above the number and to the right, thus $4^3 = 4 \times 4 \times 4 = 64$.

The number itself is the first power of the number, thus $4 = 4^1$. Also $4^2 = 4 \times 4 = 16$, or second power of 4; $4^4 = 4 \times 4 \times 4 \times 4 = 256$, or fourth power of 4. $4^5 = ?$ $3^6 = ?$

The **Cube Root** of a number is one of the three equal factors whose product is that number.

The cube root of 64 is 4; since $4 \times 4 \times 4 = 64$.

What is the cube root of 1 ? of 8 ? of 27 ? of 125 ? of 216 ? of 343 ? of 512 ? of 729 ? of 1000 ?

$\sqrt{}$ is called the radical or root sign. $\sqrt{16}$ is read the square root of 16; $\sqrt[3]{27}$ is read the cube root of 27. Read $\sqrt{64}$; $\sqrt{243}$.

A **Perfect Cube** is a number which has an integer for its cube root. Name some numbers that are perfect cubes.

In extracting the cube root, the number of digits in the root may be determined from the number of digits in the number whose cube root is to be extracted. This may be seen from the following examples :—

$1^3 = 1$. $10^3 = 1000$. $100^3 = 1{,}000{,}000$. $1{,}000^3 = 1{,}000{,}000{,}000$.

$9^3 = 729$. $99^3 = 907{,}299$. $999^3 = 997{,}002{,}999$.

From these examples it may be seen :

1. That a root of one place may have from one to three places in the cube (1 to 729).

2. That the addition of one place to the root adds three places to the cube (1,000 to 907,299).

3. The cube root of a number of three digits or less is a number of one digit (1 to 9); of a number of not more than six places or not less than four is a number of two digits (10 to 99); of a number of not more than nine digits or not less than seven is a number of three digits (100 to 999), etc.

If we point off a number into periods of three figures each, beginning at the right, the sum of the number of full periods and the one full or partial period on the left will indicate the number of periods in the cube root.

Thus, 36 | 897 | 845 has three figures in its cube root.
Also, 354 | 894 | 912 " " " " " " "

To find the cube of 64 :

$$64^3 = (60+4)^3 = (60+4)\ (60+4)\ (60+4).$$

By Multiplication :

$$\begin{array}{r} 60+4 \\ 60+4 \\ \hline 60\times 4+4^2 \\ 60^2+60\times 4 \\ \hline 60^2+2\times 60\times 4+4^2 \\ 60+4 \\ \hline 60^2\times 4+2\times 60\times 4^2+4^3 \\ 60^3+2\times 60^2\times 4+60\times 4^2 \\ \hline 60^3+(3\times 60^2\times 4)+(3\times 60\times 4^2)+4^3=(64)^3 \end{array}$$

That is, the cube of any number of two digits is equal to the cube of the tens, plus three times the square of the tens multiplied by the units, plus three times the tens multiplied by the square of the units, plus the cube of the units.

Find by this method the cubes of 25, 34, 46 and other numbers, and verify by actual multiplication.

EXTRACTION OF THE CUBE ROOT.

EXAMPLE 12. Find the cube root of 262144:

	6		262 | 144
			216
18	4	10800	46144
		736	
		11536	46144

$\sqrt[3]{262144} = 64$

1. Divide the number into two periods by a vertical line separating three figures on the right.

2. Take the nearest perfect cube not greater than 262, which is 216, and set it under the 262, and place its cube root, which is 6, in a line with 262144.

3. Substract 216 from 262, and annex the second period, 144, to the remainder.

4. Place three times the first figure of the root, 18, to the extreme left, in a line with the remainder, and three time the square of the first figure of the root, 108, with two noughts annexed, just on the left of the remainder and in a line with it.

5. Divide the remainder, 46144, by 10800 and set the quotient, 4, (the second figure of the root) midway between 18 and 10800. Read 18 4 as 184; multiply this by the 4; place the result, 736, under 10800 and add them; this gives 11536; multiply this last by 4 and put the result 46144 under the first remainder. As it is equal to the remainder the operation is complete and the required root is 64.

NOTE.—Sometimes when dividing the remainder we get a number (5) which when we carry on the operation, we find to be too large. We must then try a smaller number.

By examining Example 12 it will be observed:

1. That 216 (216000) $= 60^3$
2. That 10800 $= (3 \times 60^2)$
3. That 736 $= (3 \times 60 + 4)4 = (3 \times 60 \times 4) + 4^2$

4. That $11536\ (\text{II} + \text{III}) = (3 \times 60^2) + (3 \times 60 \times 4) + 4^2$

5. That $11536 \times 4 (= 46144) = (3 \times 60^2)4 + (3 \times 60 \times 4)4 + 4^2 \times 4 = (3 \times 60^2 \times 4) + (3 \times 60 \times 4^2) + 4^3$

6. That $216000 + 46144\ (= 262144) = 60^3 + (3 \times 60^2 \times 4) + (3 \times 60 \times 4^2) + 4^3$

EXERCISE 11.

Find the cube roots of:—

1. 12167	**4.** 103823	**7.** 132651	**10.** 704969
2. 24389	**5.** 97336	**8.** 300763	**11.** 804357
3. 42875	**6.** 1331	**9.** 421875	**12.** 456533

EXAMPLE 13. Find the cube root of 182284263 :

	5		182 | 284 | 263	
			125	
15	6	7500	57284	
		936		
		8436	50616	
		36		
168	7	940800	6668263	
		11809		
		952609	6668263	

$\sqrt[3]{182284263} = 567$

1. Separate the number into three periods and set down the nearest perfect cube not greater than 182, which is 125, and set down its cube root, 5, as before. Subtract as before and annex the next period, 284.

2. Set down 3 times 5 to the extreme left and 3 times the square of 5, with two noughts annexed, a little to the left of the remainder.

3. Divide 57284 by 7500 which gives 7. As 7 will be found to be too large try 6 for the second figure of the root, placing it midway between the 15 and 7500.

4. Read 15 6 as 156 and multiply it by the 6, which gives 936; add this to the 7500 and multiply the result by the second figure (6) of the root. Subtract the product 50616, and annex the next period.

5. Set down three times 56, which is 168, and three times the square of 56 which is 9408.

N.B.—This last result can be obtained by setting the square of

6 (the last figure of the root obtained) under the last complete divisor and adding the three numbers connected by the bracket. (The explanation of this will make an interesting and not difficult exercise.)

6. Annex two noughts to the 9408, which gives 940800. Use this for a trial divisor, which gives 7. Read 168 7 as 1687 and multiply it by 7, which gives 11809. Add; and multiply the result by 7 and place the product under last remainder.

EXAMPLE 14. Find the cube root of 130323843:

	5		130 | 323 | 843
			125
15	0	7500	5323843
150	7	750000	
		10549	
		760549	5323843

In this case the first trial divisor, 7500, is so large that 0 must be the next figure of the root since if any larger number be used, when the true divisor is found it will be too great. The next period, 843, is brought down. 50 is multiplied by 3 and the result 150 is set down under the 15. Three times the square of 50 with two noughts annexed is also set down in a line with the 150 for a new trial divisor. The remainder of the process is the same as before, the answer being 507.

EXAMPLE 15. Find the cube root of 673373097125:

	8		673 | 373 | 097 | 125
			512
24	7	19200	161373
		1729	
		20929	146503
		49	14870097
261	6	2270700	
		15696	
		2286396	13718376
		36	1151721125
2628	5	230212800	
		131425	
		230344225	1151721125

EXERCISE 12.

Find the cube root of :—

1.	2048383	6.	224755712	11.	955671625
2.	16194277	7.	270840023	12.	122615327232
3.	29503629	8.	525557943	13.	62712728317
4.	67917312	9.	746142643	14.	1076890625
5.	214921799	10.	721734273	15.	10460353203

CUBE ROOT OF DECIMALS.

Since $.1^3 = .001$, $.04^3 = .000064$, etc., it follows that the cube of a decimal of one place is a decimal of three places; the cube of a decimal of two places is a decimal of six places, etc.

Hence, when the cube root of a decimal is to be extracted, the decimal must be of the third, sixth or ninth place, etc.

EXAMPLE 16. Find the cube root of .02 to 3 places :

	2		.020 | 000 | 000
			.008
6	7	1200	12000
		469	
		1669	11683
		49	
		214700	317000
81	1	811	
		215511	215511
			101489

Answer, .271+.

Since the root is to be extracted to three places .02 is made a decimal of nine places. This is separated into periods of three places, each counting to the right from the decimal point. The nearest perfect cube not greater than .020 is .008. The root of this is 2.

The remainder of the process is the same as in whole numbers.

When a number is not a perfect cube its root can be extracted to any number of decimal places by adding periods, of three noughts each, to the remainders.

The cube root of a common fraction may be extracted by extracting the cube root of the numerator and denominator or by reducing it to a decimal and then extracting the root. Note particularly that when the cube root of any number that is a decimal, or that has a decimal as part of it, is to be extracted that the decimal places must be three or some multiple of three. When the decimal is a recurring one the repeating figures must be annexed instead of noughts.

EXERCISE 13.

Find to three places the cube roots of :—

1. 3
2. .3
3. .03
4. $\frac{64}{343}$
5. $\frac{1000}{2744}$
6. $\frac{1}{9}$
7. 123
8. 446
9. 900
10. .12345678
11. 1.234567
12. 7
13. $49.\dot{2}9\dot{6}$
14. .0001
15. $60.\dot{8}$
16. $171.\dot{9}$

EXAMINATION PAPER No. 1.

1. A man had a lot of eggs. He sold $\frac{1}{3}$ of them at 10 cents a dozen, $\frac{1}{5}$ at 12 cents and 12 dozen at 15 cents. He had 31 eggs left which were spoiled. What did he get for his eggs?

2. What is the square root of $\frac{3}{70}$, correct to 4 places?

3. Find the value of $4\sqrt{.005}$.

4. $\frac{5}{6}$ is $3\frac{1}{4}\%$ of what number?

5. Multiply £5 19s.$11\frac{1}{4}$d. by $14\frac{3}{11}$.

EXAMINATION PAPER No. 2.

1. Simplify $\dfrac{\$\frac{3}{4} \times .05 \text{ of } 1\frac{1}{2}}{6 - \frac{2}{3} \text{ of } (5 - .5 \times 2)}$.

2. What is the diameter of a circular field containing 636174 square rods?

3. Find the square root of .01170724.

4. What is the weight of a piece of granite 9 ft. 4 in. long, 2 ft. 3 in. wide, and 1 ft. 3 in. thick? (Sp. gr. of granite is 2.72).

5. What is the specific gravity of a substance that weighs 10% more in air than in water?

EXAMINATION PAPER No. 3.

1. What per cent. of a pound avoir. is a pound troy?

2. What per cent. of an ounce avoir. is an ounce troy?

3. Simplify $6.03\dot{8} + .0\dot{8}7\dot{5} - 4.00\dot{3}7\dot{6}$.

4. What is the value of £$.\dot{4}5 \times .3\dot{9} \div .5\dot{4}\dot{5}$?

5. What is the cost of papering the walls of a room 16 ft. long, 15 ft. 4 in. wide and 9 ft. 6 in. high, with paper 20 in. wide, 7 yds. in a roll, at 18 cents a roll?

EXAMINATION PAPER No. 4.

1. What will it cost to lay a plank floor in a drill shed 200 ft. long and 120 ft. wide, the plank being $2\frac{1}{2}$ in. thick, and 8 cents a foot?

2. A school room is 32 ft. long and 24 ft. wide. What is the distance from a corner near the floor to the corner farthest away near the ceiling, ceiling 10 ft. high?

3. What will it cost to carpet a room 14 ft. by 12 ft. 4 in. with carpet 27 in. wide and $1.75 a yard?

4. One side of a triangular field is 220 chains. A line drawn from the opposite angle at right angles to this side measures 200 chains. How many acres in the field?

5. Simplify $3\sqrt{.04\dot{5}} \times 4$.

EXAMINATION PAPER No. 5.

1. How many cubic feet in a globe whose diameter is 24 inches?

2. Find the cube root of $\frac{49}{72}$ to four places.

3. A cistern is 20 ft. long, 10 ft. wide and 8 ft. deep. What is the area of the bottom of a cubical vessel that will hold as much?

4. Write the value of $(.\dot{9})^3$ using as few figures as possible.

5. Multiply $.\dot{5}4\dot{9}$ by 729 without reducing $.\dot{5}4\dot{9}$ to a common fraction. Prove your answer by reducing $.\dot{5}4\dot{9}$ to a common fraction and multiplying.

EXAMINATION PAPER No. 6.

1. By selling wheat at 5s. 6d. a bushel I gained $37\frac{1}{2}\%$; what per cent. should I have gained if I had sold at 6s. 6d.?

2. Simplify $(3.6\dot{3} \times .08\dot{2}) + (3.0\dot{6} - 2.7\dot{1}\dot{9})$.

3. Find the value of .125 of an acre.

4. Bought 60 lbs. of tea at 20 cents a lb., 100 lbs. at 25 cents and 40 lbs. at 30 cents. At how much per pound must I sell the mixture to gain $13?

5. From 7 fur., 39 rods, 5 yds., 2 ft., 7 in. take 1 mi.

SCALES.

SIMPLE OR REGULAR SCALES.

If a large number of pencils which are heaped promiscuously in a room are to be counted and prepared for shipment, it may be done by tying them up, 10 in a bundle, and binding the bundles into packages of 10 bundles in each package, and placing the packages, 10 in a box, and packing the boxes in bales of 10 boxes each.

If on tying the pencils into bundles of 10 each, we have 1329 bundles and 7 *pencils* over,

We get $\frac{1329}{10}$, or 132 packages and 9 bundles over,

And $\frac{132}{10}$, or 13 boxes and 2 *packages* over,

And $\frac{13}{10}$, or 1 bale and 3 *boxes* over;

Or 1 *bale*, 3 *boxes*, 2 *packages*, 9 *bundles*, 7 *pencils*.

Now, as 10 pencils make a bundle, 10 bundles 1 *package*, etc., and in the Common or *Decimal* system of Notation, 10 *ones* make 1 *ten*, 10 *tens* make 1 *hundred*, etc., the 1 *bale*, 3 *boxes*, 2 *packages*, 9 *bundles*, 7 pencils, correspond to 1 *ten-thousand*, 3 *thousands*, 2 *hundreds*, 9 *tens*, 7 *ones;* or 13297 *ones*.

But the pencils may be tied not only in systems of 10, but in systems of 9, 8, 7, 6, 12 or any other number. Now, if they are tied in systems of 8, we have $\frac{13297}{8}$, or 1662 bundles and 1 pencil over,

And 1662 bundles $=\frac{1662}{8}$, or 207 packages and 6 bundles over,

And 207 packages $=\frac{207}{8}$, or 25 boxes and 7 packages over,

And 25 boxes $=\frac{25}{8}$, or 3 bales and 1 box over;

Or 3 bales, 1 box, 7 packages, 6 bundles, 1 pencil; that is 31761 in the *eight* or *Octonary* system.

Again, if we take 13297 pencils as expressed in the decimal system, and collect them into a system of 7 each, we have $\frac{13297}{7}$, or 1899 bundles and 4 pencils over,

And 1899 bundles $=\frac{1899}{7}$, or 271 packages and 2 bundles over,

And 271 packages $=\frac{271}{7}$, or 38 boxes and 5 packages over,

38 boxes $=\frac{38}{7}$, or 5 bales and 3 boxes over;

Or 5 bales, 3 boxes, 5 packages, 2 bundles, 4 pencils; that is 53524 in the *seven* or *Septenary* system.

This method is a universal one, and will be hereafter referred to as the *Universal* or *Division* rule.

Again, reversing the process:—

5 bales, 3 boxes $= 5 \times 7 + 3$, or 38 boxes,
38 boxes, 5 packages $= 38 \times 7 + 5$, or 271 packages,
271 packages, 2 bundles $= 271 \times 7 + 2$, or 1899 bundles,
1899 bundles, 4 pencils $= 1899 \times 7 + 4$, or 13297 pencils.

Or thus, the two processes in the two usual arithmetical methods, —the *division* and the *multiplication* methods respectively.

7 | 13297 pencils.
7 | 1899 bundles + 4 pencils over.
7 | 271 packages + 2 bundles over.
7 | 38 boxes + 5 packages over.
5 bales + 3 boxes over.

Ans: 5 bales, 3 bo. 5 pk. 2 bu. 4 pen.

And: 5 ba. 3 bo. 5 pk. 2 bu. 4 pen.

7
38 boxes.
7
271 packages.
7
1899 bundles.
7
13297 pencils.

Thus we see that in *Notation* there may be different *Scales* or ratios by which numbers increase or decrease.

Of course the ordinary scale of notation for whole numbers is the *decimal*, but it is possible to express numbers in other scales.

In any scale of notation the number of *ones* required to make one of the next higher order is called the **Radix** of the scale.

When the radix does not vary, the scale is said to be **Regular** or **Simple** but when the radix changes, as in the case of the table of sterling money, the scale is called **Irregular** or **Compound.**

In scales, other than the decimal, the radix is indicated by a small figure written below. Thus, 7246_8 means 7246 in the octonary scale.

SOME REGULAR OR SIMPLE SCALES.

Name.	*Radix.*	*Name.*	*Radix.*
Binary	2.	Nonary	9.
Ternary	3.	Denary or Decimal	10.
Quaternary	4.	Undenary	11.
Quinary	5.	Duodenary or Duodecimal	12.
Senary	6.	Vigesimal	20.
Septenary	7.	Sexagesimal	60.
Octonary	8.		

NOTATION.—REGULAR SCALE.

When numbers are expressed in any uniform scale, it is necessary to employ as many characters as there are ones in the radix of the scale, and one of these must be 0.

In expressing numbers in scales higher than the decimal we use a single symbol such as t for 10, e for 11, etc.

As the terms *tens, hundreds*, etc., are always used in reference to the decimal scale, numbers in other scales must be read by naming the number of units in each order.

Thus 243 in the *quinary* scale is read thus: *Quinary scale, 2 ones of the third order, 4 ones of second, and 3 of the first.*

EXAMPLE.—Write the numbers from 1 to 12 in the quaternary scale.

Since 4 ones of any order are equal to 1 of the next higher, the characters we use are 1, 2, 3, 0, and the numbers from 1 to 12 would be set down 1, 2, 3, 10, 11, 12, 13, 20, 21, 22, 23, 30. In the common number 9 for instance there are 2 *fours* and 1 over. So 9 in the decimal scale is written 21 in the quaternary.

EXAMPLE.—Write 13297 of the decimal scale in the quinary.

5 | 13297
5 | 2659 and 2 over
5 | 531 and 4 over
5 | 106 and 1 over
5 | 21 and 1 over
4 and 1 over

By dividing by 5 we get 2659 *ones* of the *second* order and 2 of the *first* order remaining. The second time we divide by 5 we get 531 *ones* of the *third* order, and 4 *ones* of the *second* order over. The last time we divide we get 4 *ones* of the *sixth* order and 1 of the *fifth* order remaining. So the number in the quinary scale is 411142.

EXAMPLE.—Change 134052 from the senary scale to the decimal, and also 201210 from the ternary.

Use the multiplication method, which is the more convenient when the reduction is to the decimal scale.

134052 senary.	201210 ternary.
6	3
9	6
6	3
58	19
6	3
348	59
6	3
2093	178
6	3
12560 decimal.	534 decimal.

EXAMPLE.—Solve the same two problems by the ***division*** or ***universal*** method.

10 | 134052 senary scale to the decimal.
10 | 5452 and 0 over
10 | 325 and 6 over
10 | 20 and 5 over
10 | 1 and 2 over
0 and 1 over. Ans. 12560.

10 | 201210 ternary scale to the decimal.
10 | 1222 and 4 over
10 | 12 and 3 over
0 and 5 over. Ans. 534.

EXERCISE 14.

1. Express in the octonary scale 5706, 7134, 6544.
2. " " quinary scale 4321, 3042, 34012.
3. " " nonary scale 7238, 77106, 88134.
4. " " undenary scale 5137, 8369, 75903.
5. " " duodecimal scale 13748, 78645, 70863.
6. " " senary scale 25034, 111205, 340251.
7. " " decimal scale 42031_5, 123456_7.
8. " " " " 71483_9, 71542_8, 10101_2.
9. " " " " $9t34_{11}$, 58627_9, 210102_3.
10. " " " " 37054_8, $3819e_{12}$, $268te_{12}$.

In arithmetical processes in any scale it is only necessary to remember what the particular scale is. All numbers equal to, or greater than the radix, are divided by the radix, and the remainder set down.

Numbers in *different* scales must be reduced to *the same scale* before any arithmetical operation is attempted. In multiplying or dividing numbers not in the decimal scale, time will probably be saved by reducing them first to the decimal scale, and then back again to the required scale after the operation has been performed.

EXERCISE 15.

1. Add 4253_6, 12345_6, 40231_6, 52430_6.
2. " 58727_9, 80062_9, 5374_9, 38144_9.
3. " 4322_5, 12340_5, 43210_5, 33442_5.
4. " 58471_{11}, $89t44_{11}$, 77692_{11}, $t3729_{11}$.
5. Multiply 7726_8 by 543_8.
6. " 70843_9 by 6082_9.

7. Subtract 3758946_{11} from 8881342_{11}.
8. " 1234504_{6} from 5403210_{6}.
9. " 5638927_{12} from 7783924_{12}.

[As the weights and measures of the metric system are all in the Regular or Simple Scale, we give the tables of the metric system here.]

TABLES OF DENOMINATE NUMBERS IN THE DECIMAL (A SIMPLE OR REGULAR) SCALE.

METRIC SYSTEM.

MEASURES OF LENGTH.

10 MILLIMETERS (mm.)	= 1 **centimeter** (cm.)	= .01 of a meter.
10 centimeters	= 1 decimeter (dm.)	= .1 " "
10 decimeters	= 1 **meter** (m.)	= 39.3708 + inches.
10 meters	= 1 dekameter (Dm.)	= 10 meters.
10 dekameters	= 1 hektometer (Hm.)	= 100 "
10 hektometers	= 1 **kilometer** (Km.)	= 1000 "
10 kilometers	= 1 myriameter (Mm.)	= 10000 "

MEASURES OF SURFACE.

100 square millimeters (qmm.)	= 1	square	**centimeter** (qcm.)
100 " centimeters	= 1	"	decimeter (qdm.)
100 " decimeters	= 1	"	**meter** (qm.
100 " meters	= 1	"	dekameter (qDm.)
100 " dekameters	= 1	"	hektometer (qHm.)
100 " hektometers	= 1	"	**kilometer** (qKm.)

In measuring land the square *meter* is called a CENTAR. A square each side of which is 10^m long is called an AR; and a square having each side 100^m long is called a HEKTAR.

100 centars (ca.)	= 1 ar (a.)
100 ars	= 1 hektar (Ha.)

MEASURES OF VOLUME.

1000 cubic millimeters (cu. mm.)	= 1	cubic	**centimeter** (cu. cm.)
1000 " centimeters	= 1	"	decimeter (cu. dm.)
1000 " decimeters	= 1	"	**meter** (cu. m.)

In measuring wood the cubic meter is called a *ster* (st.); one-tenth of a cubic meter is a *decister* (dst.); and ten cubic meters is called a *dekaster* (Dst.).

MEASURES OF CAPACITY.

10 milliliters (ml.) = 1 **centiliter** (cl.)
10 centiliters = 1 deciliter (dl.)
10 deciliters = 1 **liter** (l.)
10 liters = 1 dekaliter (Dl.)
10 dekaliters = 1 hektoliter (Hl.)
10 hektoliters = 1 **kiloliter** (Kl.) = 1 cu m.

MEASURES OF WEIGHT.

10 *milligrams* (mg.) = 1 *centigram* (cg.)
10 centigrams = 1 decigram (dg.)
10 decigrams = 1 **gram** (g.) = 15.432348 grains.
10 grams = 1 dekagram (Dg.)
10 dekagrams = 1 hektogram (Hg.)
10 hektograms = 1 **kilogram** (Kg.)
1000 kilograms = 1 metric ton (T.)

EQUIVALENTS IN OTHER MEASURES.

1 meter = 39.37 + in., or 1.0936 + yd.
1 kilometer = .62137 + mi., or about $\frac{5}{8}$ of a mile.
1 square meter = 1.196 + sq. yd., or about $10\frac{3}{4}$ sq. ft.
1 ar = 119.6 + sq. yd., or about 4 sq. rd.
1 hektar = 2.471 + acres.
1 cubic meter = 1.308 + cu. yd., or about $35\frac{1}{3}$ cu. ft.
1 liter = .8804 + imperial quarts, or about $1\frac{3}{4}$ pints.
1 hektoliter = 2.751 + bushels.
1 gram = 15.432 + grains.
1 kilogram = 2.2046 + lb., or about $2\frac{1}{5}$ lb. av.
1 metric ton = 2204.6 + lb. or about 1 ton, 200 lb.
1 yard = .9144^{m} +. 1 mile = 1.60935^{Km}.
1 quart = 1.136^{l} +. 1 bushel = .3635$^{Hl.}$ +.
1 lb. = .4536^{Kg}. 1 ster = .2759 cord.

The Associated Chambers of Commerce of the British Empire at their convention held in London, June, 1896, at which thirty-two Canadian delegates were present, passed the following resolution:

WHEREAS, The British system of weights and measures, which vary constantly in every part of the British Empire, is a source of constant annoyance, loss of time, and a formidable obstacle to local, imperial and foreign trade; and whereas the metric system has now been universally recognized as the most perfect decimal system, and it is generally adopted by nations of both continents, with the exception of the British Empire and United States of America, therefore, be it

Resolved, That the metric system of weights and measures be adopted without further delay by the several governments of the British Empire, the yard being extended to the meter, the quart to the liter, and the two pound weight to the kilogram.

DENOMINATE NUMBERS IN IRREGULAR OR COMPOUND SCALES.

When the scale changes with each order of figures in the number, the scale is called an *Irregular* or *Compound Scale.*

Thus in the case of the pencils of which we had been speaking, instead of having the same number of pencils in each bundle as there are of bundles in each package, etc., we might use different numbers for each denomination. For instance, we might put 4 pencils in each bundle, 12 bundles in each package and 20 packages in each box. In that case the 13297 pencils would give:

$\frac{13297}{4}$, or 3324 bundles and 1 pencil over,

And 3324 bundles $=\frac{3324}{12}$, or 277 packages,

And 277 packages $=\frac{277}{20}$, or 13 boxes, 17 packages over; that is 13 boxes, 17 packages, 0 bundles and 1 pencil.

In the same way, if we take 13297 farthings (sterling money) and put them up in 4's or penny bundles, we have $\frac{13297}{4}$, or 3324 pence and 1 farthing over,

And 3324 pence $=\frac{3324}{12}$, or 277 shillings,

And 277 shillings $=\frac{277}{20}$, or 13 pounds and 17 shillings over; that is £13, 17s. 0d. 1 far.

Or thus by the *universal* method:

4 | 13297 far.
12 | 3324d., 1 far.
20 | 277s.
£13 17s. 0d. 1 far.

And reversing the process by the *multiplication* method:

£.	S.	D.
13	17	$0\frac{1}{4}$
20		
277s.		
12		
3324d.		
4		
13297 far.		

Or by the *Universal Method*:—

	£.	S.	D.	
10	13	17	$0\frac{1}{4}$	
10	1	7	$8\frac{1}{4}$	and 7 *over.*
10	0	2	$9\frac{2}{4}$	and 9 *over.*
10	0		$3\frac{1}{4}$	and 2 *over.*
10	0		$0\frac{1}{4}$	and 3 *over.*
	0			and 1 *over.*

Ans. 13297 farthings.

Again, if we take 850617 drams, the lowest denomination of

common (avoirdupois) weight, and put them up in 16-dram or *ounce* packages, we get $\frac{53163}{16}$, or 53163 ounces and 9 drams over.

And 53163 ounces put up in 16-ounce or *pound* packages, give $\frac{3322}{16}$, or 3322 pounds and 11 ounces over,

And 3322 pounds put up in 100-pound or *hundredweight* boxes, give $\frac{3322}{100}$, or 33 hundredweight and 22 pounds over.

And 33 hundredweight put up in 20-hundredweight or *ton* boxes, give $\frac{33}{20}$, or 1 ton and 13 hundredweight; that is, 850617 dr. = 1 ton, 13 cwt. 22 lb. 11 oz. 9 dr.

Or thus, *ascending to higher denominations* (universal method);

16 | 850617 dr.
16 | 53163 oz. 9 dr.
100 | 3322 lb. 11 oz.
20 | 33 cwt. 22 lb.
1 t. 13 cwt. 22 lb. 11 oz. 9 dr.

And thus, *descending to lower denominations* (multiplication method):

1 t. 13 cwt. 22 lb. 11 oz. 9 dr.
20
33 cwt.
100
3322 lb.
16
19943
3322
53163 oz.
16
318987
53163
850617 dr.

Or by the *Universal Method*:—

	ton.	cwt.	lb.	oz.	dr.	
10 |	1	13	22	11	9	
10 |	3	32	4	5		and 7 *over.*
10 |	33	3	10			and 1 *over.*
10 |	3	5	2			and 6 *over.*
10 |	5	5				and 0 *over.*
		8				and 5 *over.*

Ans. 850617 drams.

Here in the ordinary operation given above of reducing 850617 dr. to tons, etc., we proceeded as in the simple scale, beginning with the radix of the lowest order, 16 drams = 1 ounce, and following with

the other radices, 16 oz. = 1 lb.; 100 lb. = 1 cwt.; 20 cwt. = 1 ton. We thus showed that 850617 drams in the simple or regular decimal scale are equal to 1 ton, 13 cwt. 22 lb. 11 oz. 9 dr. in the "Compound" Avoirdupois scale.

But we have also shown, by the exercises done by the *universal method*, that the same division rule will change the notation from pounds to farthings, from tons to drams, or from one scale to another.

In this universal method of *reduction*, as will be seen from the examples, we *divide the number in the given system by the radices of the required system, in order, beginning with the radix of the lowest order, and the remainders in similar order will be the number in the required system.*

But the more convenient method, when reducing from a compound to a simple scale, is the ordinary one of multiplication. In this, as has been observed in reducing 1 ton, 13 cwt., etc., to drams, we began *by multiplying the highest order in the given number by the radices, in order, beginning with the radix of the highest order, and adding in each lower order in its proper place.*

TABLES OF DENOMINATE NUMBERS IN THE IRREGULAR OR COMPOUND SCALES.

MEASURES OF LENGTH.

12 inches (in.)	=	1 foot (ft.)
3 feet	=	1 yard (yd.)
5½ yards	=	1 rod (rd.)
40 rods	=	1 furlong (fur.)
8 furlongs	=	1 mile (mi.)

1 mi. = 320 rd. = 1760 yd. = 5280 ft. = 63,360 in.

A *hand* = 4 in. A *span* = 9 in. A *common cubit* = 18 in. A *sacred cubit* = 21.888 in. A *fathom* = 6 ft. A *knot*, a *nautical mile*, or a *geographic mile* = 6086.7 ft., or $\frac{1}{60}$ of $\frac{1}{360}$ of the circumference of the earth.

The standard yard is fixed by dividing a pendulum which vibrates seconds in a vacuum at 62° F. in the latitude of London and at the level of the sea, into 391,393 equal parts, and taking 360,000 of these parts for the yard.

SURFACE MEASURES.

144	square inches (sq. in.)	=	1	square foot (sq. ft.)
9	" feet	=	1	" yard (sq. yd.)
30¼	" yards	=	1	" rod (sq. rd.)
160	" rods	=	1	acre (A.)
640	acres	=	1	square mile (sq. mi.)

1 sq. mi. = 640 A. = 102,400 sq. rd. = 3,097,600 sq. yd. = 27,878,400 sq. ft. = 4,014,489,600 sq. in.

Land surveyors use a *chain* 4 rods or 66 ft. long, divided into 100 links. A *link*=7.92 in. 10 sq. chains=1 acre. 80 chains=1 mile.

An engineer's chain is 100 ft. long and consists of 100 links.

Shingling, roofing, etc., are commonly estimated by the *square*. Each side of the *square* is 10 ft. long and equal to 100 sq. ft.

A square piece of land measuring 70 yds. on each side contains very nearly one acre.

MEASURES OF VOLUME.

1728 cubic inches = 1 cubic foot (cu. ft.)
27 " feet = 1 " yard (cu. yd.)

A *cord* of wood or stone is a pile 8 ft. long, 4 ft. wide, and 4 ft. high. A *perch* of stone or masonry is 16½ ft. long, 1½ ft. thick, and 1 ft. high, and contains 24¾ cu. ft. A *cubic yard* of earth is considered a *load*.

MEASURES OF CAPACITY.

2 pints (pt.) = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
2 gallons = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

The legal or "Imperial" bushel of Canada contains 8 imperial gallons, and 1 imperial gallon contains 277.274 cubic inches, or 10 lb. of distilled water, temperature 62° F., the barometer standing at 30 inches. Our *Imperial bushel* equals 2218.192 cu. in., and the *Winchester bushel* used in the United States equals 2150.42 cu. in.

The "Weights and Measures" Act of the Canadian Parliament of 1873 fixed the number of pounds to the bushel of each article named below as follows:—

Oats	34 lb.	Flax Seed	50 lb.	Beans	60 lb.
Barley	48 "	Corn	56 "	Peas	60 "
Buckwheat	48 "	Rye	56 "	CloverSeed	60 "
Timothy Seed	48 "	Wheat	60 "	Potatoes	60 "

MEASURES OF WEIGHT.

AVOIRDUPOIS WEIGHT.

16 drams (dr.) = 1 ounce (oz.)
16 ounces = 1 pound (lb.)
100 pounds = 1 hundred weight (cwt.)
20 hundred weight = 1 ton (T.)
A long ton = 2240 lbs.
(1 T. = 20 cwt. = 2000 lbs. = 32000 oz. = 512000 dr.)

Everything except precious metals, jewels and medicines is weighed by avoirdupois weight.

TROY OR JEWELLERS' WEIGHT.

24 grains (gr.)	=	1 pennyweight (dwt.)
20 pennyweights	=	1 ounce (oz.)
12 ounces	=	1 pound (lb.)

In weighing diamonds 1 carat = $3\frac{1}{8}$ troy grains.

APOTHECARIES' OR DRUGGISTS' WEIGHT.

20 grains (gr.)	=	1 scruple (sc. or ℈)
3 scruples	=	1 dram (dr. or ℥)
8 drams	=	1 ounce (oz. or ℥)
12 ounces	=	1 pound (lb.)

The *grain*, *ounce*, and *pound* are identical in weight with the troy *grain*, *ounce*, and *pound*.

1 lb. avoirdupois = 7000 gr. 1 lb. $\left\{\begin{matrix}\text{troy and}\\\text{apothecaries.}\end{matrix}\right\}$ = 5760 gr.

1 oz. " = $437\frac{1}{2}$ gr. 1 oz. " = 480 gr.

144 lb. av. = 175 lbs. ap. or troy.

Drugs are bought and sold by avoirdupois weight although compounded by apothecaries'.

MEASURE OF TIME.

60 seconds (sec.)	=	1 minute (min.)
60 minutes	=	1 hour (hr.)
24 hours	=	1 day (da.)
7 days	=	1 week (wk.)
365 "	=	1 common year.
366 "	=	1 leap "

Centennial years exactly divisible by 400, and other years exactly divisible by 4, are *leap years*.

The civil day begins and ends at midnight.

The earth revolves around the sun in 365 da., 5 hr., 48 min., 49.7 sec.

CIRCULAR OR ANGULAR MEASURE.

60 seconds (″)	=	1 minute (′)
60 minutes	=	1 degree (°)
360 degrees	=	1 circumference.

An angle of 90° is called a *right angle*, and an arc of 90° is called a *quadrant*.

MISCELLANEOUS TABLE.

12 units	=	1 dozen.
12 dozen	=	1 gross.
12 gross	=	1 great gross.
20 units	=	1 score.
196 lb. of flour	=	1 barrel.
200 lb. pork or beef	=	1 barrel.
100 lb. dried fish	=	1 quintal.
24 sheets of paper	=	1 quire.
20 quires	=	1 ream.
2 reams	=	1 bundle.
5 bundles	=	1 bale.

BOOKS.

A book formed of sheets folded

in 2	leaves	is a folio.
in 4	"	a quarto or 4to.
in 8	"	an octavo or 8vo.
in 12	"	a 12mo.
in 16	"	a 16mo.
in 18	"	an 18mo.
in 24	"	a 24mo.
in 32	"	a 32mo.

ENGLISH MONEY.

4 farthings (far.)	=	1 penny (d.)
12 pence	=	1 shilling (s.)
20 shillings	=	1 pound (£.)

The Canadian gold coins are the British Sovereign, worth $4.86\frac{2}{3}$, and the British half-sovereign.

The Canadian silver coins are the 50-cent piece, the 25-cent piece, the 10-cent piece and the 5-cent piece.

Our 1-cent piece, made of bronze, is one inch in diameter, and 100 cents weigh one pound.

The gold coins of the United States are Eagle ($10), Double Eagle, Half Eagle, Quarter Eagle and Dollar.

The British gold coin is $\frac{11}{12}$ pure metal and $\frac{1}{12}$ alloy.

The gold and silver coinage of the U. S. is $\frac{9}{10}$ pure.

In Canada and Great Britain the silver coin is $\frac{37}{40}$ silver and $\frac{3}{40}$ copper.

Gold and silver alloyed as in the coinage is called *standard*.

Standard gold is $\frac{22}{24}$ pure, or 22 *carats* fine. When pure gold is alloyed with 6 parts of copper or other metal in 24 parts, it is then only 18 carats fine and is known as jewellers' gold.

Gold and silver before being coined is called *bullion*, and after being coined *specie*.

The relations between Canadian money and the moneys of other countries vary slightly from time to time, but the following table is substantially correct and can be used as a working basis in ordinary arithmetical operations :—

Country.	Monetary Unit.	Value in Canadian Money.	Divisions of Units.
Argentine Rep.	Peso	$0.965	100 centavos = 1 peso.
Austria	Florin	.336	100 kreutzers = 1 florin.
Belgium	Franc	.193	100 centimes = 1 franc.
Bolivia	Boliviano	.68	100 centavos = 1 boliviano.
Brazil	Milreis	.546	1000 reis = 1 milreis.
Chili	Peso	.912	100 centavos = 1 peso.
Cuba	Peseta	.926	100 centimos = 1 peseta.
Denmark	Crown	.268	100 ore = 1 crown.
Ecuador	Sucre	.68	100 centavos = 1 sucre.
Egypt	Pound	4.943	100 piastres = 1 pound.
France	Franc*	.193	100 centimes = 1 franc.
German Empire.	Mark	.238	100 pfennig = 1 mark.
Great Britain	Pound Sterling	4.86⅔	20 shillings = 1 pound.
Greece	Drachma	.193	100 lepta = 1 drachma
India	Rupee	.323	16 annas = 1 rupee.
Italy	Lira	.193	100 centesimi = 1 lira.
Japan	Yen { Gold.	.997	100 sens = 1 yen.
	Yen { Silver.	.734	
Mexico	Dollar	.739	100 centavos = 1 dollar.
Netherlands	Florin	.402	100 cents = 1 florin.
Nor'y and Sweden	Crown	.268	100 ore = 1 crown.
Peru	Sol	.68	100 centavos = 1 sol.
Portugal	Milreis	1.08	1000 reis = 1 milreis.
Russia	Rouble	.544	100 copecks = 1 rouble.
Spain	Peseta	.193	100 centimos = 1 peseta.
Switzerland	Franc	.193	100 centimes = 1 franc.
Turkey	Piastre	.044	40 paras = 1 piastre.
United States	Dollar	1.00	100 cents = 1 dollar.
Venezuela	Bolivar	.136	100 centimos = 1 bolivar.

*The *franc* is a silver coin, 1 centimeter in diameter, and weighs 1 gram.

The specific gravity of some common substances is as follow :—

Air	.001292	Ivory	1.91
Aluminum	2.56	Lead	11.3
Brass (average)	7.611	Marble (average)	2.73
Brick (common)	2.	Mercury	13.6
" (pressed)	2.4	Milk	1.032
Carbon, charcoal	1.7	Nickel	8.8
" graphite	2.3	Oak	.84
" diamond	3.5	Olive Oil	.915
Coal, soft (average)	1.25	Pine, dry	.4
" hard "	1.5	Platinum	21.5
Copper	8.9	Porcelain	2.38
Cork	.29	Quartz	2.65
Deal (average)	.66	Salt	2.13
Ether	.715	Silver	10.5
Glass, crown	2.5	Sulphur	2.5
" flint	3.33	Sulphuric Acid	1.841
Gold	19.3	Tin	7.3
Granite	[illegible]	Turpentine	.869
Ice	[illegible]	Water, sea	1.026
Iron, cast	7.[illegible]	Zinc	7.1
" wrought	7.77		

DENOMINATE NUMBERS.

EXERCISE 16.

Reduce to higher denominations :

1. 69439 farthings.
2. 718096 grains Troy.
3. " " Apothecaries'.
4. 3069544 drams.
5. 6875103 inches.
6. 70301 pints.
7. 5103876 seconds.
8. 789247^{mm}.
9. 246078 cu. in.
10. 197815 sq. in.

Reduce to lowest denomination :

11. £45 16s. $10\frac{1}{2}$d.
12. 24 y. 127da. 18hr. 44min.
13. 4.0371^{Km}.
14. 29 cwt. 37 lb. 15 oz.
15. 18 bu. 3 pk. 7 qt.
16. 4.0371^{qKm}.
17. 29 mi. 139 rd. 12 ft.
18. 20 lb. 10 oz. 6 dwt. 22 gr.
19. 20lb. 10oz. 2dr. 2scr. 6gr.
20. 2 A. 68 sq. rd. 30 sq. yd.

EXERCISE 17.

1. From the sum of 3 T. 17 cwt. 29 lb., 18 cwt. 4 lb. 9 oz., 78 lb. 17 oz. 9 dr., 3 T. 12 cwt. 51 lb. 8 oz. 11 dr. take 4 T. 92 lb. 8 dr.

2. Multiply the sum of 18 mi. 15 rd. 4 yd. 2 ft., 87 mi. 6 fur. 32 rd., 1 mi. 500 yd. 10 in. by 39.

3. Divide the difference of 19 sq. mi. 480 A. 58 sq. rd. 6 sq. yd. 75 sq. in. and 5 sq. mi. 511 A. 27 sq. yd. 4 sq. ft. 88 sq. in. by 25.

4. Express in farthings £36 11s. $3\frac{1}{4}$d.; and in pounds, etc., 23083 farthings.

5. Express in Troy weight 2 cwt. 45 lb.; and also 1 ton 7 cwt. 14 lb.

6. Express in avoirdupois weight 1 lb. 4 oz. 8 dwt. 3 gr.; and also 388 lb. 3 oz. 7 dwt. 12 gr.

7. When the sum of £241 19s. $7\frac{3}{4}$d. is divided into 67 equal parts what is the amount of each part?

8. How many allotments of 83 sq. rd. 7 sq. ft. 81 sq. in. can be made from a piece of ground of 56 A. 7 sq. rd., and what will remain?

9. Divide 17 T. 15 cwt. 53 lb. 2 oz. 3 dr. into 73 equal parts.

10. Multiply 3 mi. 3 fur. 11 rd. 1 yd. 8 in. by $\frac{4}{7}$.

11. In 65296108 sq. in. how many acres, etc.?

12. How many minutes from 14.20 o'clock, June 24th, 1896, to 8.40 o'clock, January 3rd, 1901?

13. Two persons start at the same time from places 120 mi. apart, and travel towards each other. After one travels $\frac{2}{9}$ and the other $\frac{3}{7}$ of the distance, how far are they apart?

14. The moon makes the circuit of the earth in 29 da. 12h. 44 min. 3 sec., and the earth revolves around the sun in 365 da. 5 h. 48 min. 49.7 sec. How many revolutions does the moon make while the earth makes one?

15. The aggregate weight of 123 loads of hay is 57 T. 19 cwt. 42 lb. 14 oz. What is the average weight of a load?

16. How many silver coins, each weighing $412\frac{1}{2}$ gr., can be coined from a bar of silver weighing 8 lb. 4 oz. av.?

17. When a steamer is going 20 knots an hour and a freight train 23 miles an hour, which is going the faster, and how much?

18. In water 43 fathoms deep, how many feet and inches will have to be added to a line $168\frac{1}{2}$ cubits long so that it may reach the bottom?

19. Divide 12 lb. 11 oz. 7 dr. 19 gr. by 11, and 21 lb. 2 oz. 6 gr. by 17, and add the results.

20. Beginning on Friday, March 1st, a grocer sold during the month 1 T. 9 cwt. 18 lb. 8 oz. of butter. What was his average daily sale?

[*For Explanation see Part III., Examples 1-6.*]

EXERCISE 18.

1. Express 5 cwt. 2 qr. 14 lb. in tons.

2. Express .778125 T. in lower denominations.

3. Reduce 13^{a} sq. rd. 167 sq. ft. 72 sq. in. to the decimal of an acre.

4. What is the value of .5555 of 1 lb. ap.?

5. What fraction of an ounce is $\frac{1}{1728}$ of 1 cwt.?

6. What is the value of $\frac{7}{9}$ of $\frac{8}{5}$ of £1 16s $8\frac{1}{2}$d.?

7. What is the sum of .7 rd. + .625 yd. + .713 ft. + .91 in.

8. Express 2 da. 4 hr. as a decimal of 3 wk. 3 da.

9. What will 3 T. 6 cwt. 27 lb. of coal cost at $4.75 a ton?

10. If a grocer's scales give only $15\frac{1}{4}$ oz. for a pound, of how much money does he defraud his customers in the sale of 6 bbls. of sugar, each weighing 276 lb., at 5 cents a pound?

11. Express in inches the sum of 1 knot, 7 fathoms, 8 chains, 50 links, 12 cubits and 6 hands.

12. Find, in pounds av., the total weight of 2 bbls. of flour, 3 bbls. of beef, 27 bushels of wheat, 15 bushels of

barley, 12 bushels of oats, 170 pounds of gold and 5 pounds of silver.

13. A stationer bought 6 reams of paper at $3.25 a ream, and sold $\frac{2}{3}$ of it at 25 cents a quire and the remainder in sheets at the rate of 10 cents per half dozen. What did he gain?

14. Add $\frac{2}{3}$ of an acre, $\frac{2}{5}$ of a sq. rod, $\frac{5}{8}$ of a sq. yd., and $\frac{7}{12}$ of a sq. ft.

15. Add together .7956 of a week, 1.562 of a day, and .79 of an hour, and express the sum in days.

[*For Explanation see Part III., Examples 7-12.*]

EXERCISE 19.

1. Express 178 yd. 2 ft. 5 in. in meters.

2. Express 391.008^{m} in yards, feet and inches.

3. In 273.4 yd. how many meters?

4. Reduce 666.8262^{m} to yards.

5. In 55 mi. 5 fur. 15 rd. 5 yd. 2.28 in. how many kilometers?

6. Express 831.978 quarts in liters.

7. Express 454.4^{l} in gallons.

8. Express 113.73^{Hl} in bushels, etc.

9. What is the cost of digging a cellar 8.4^{m} long, 6.5^{m} wide and 2.5^{m} deep, at $1.50 per cubic meter?

10. A cask capable of holding a metric ton of water is filled with oil. What is the oil worth at 25 cents per gallon?

11. When a piece of matting 2 yards square costs $1, what is the cost per square meter?

12. Express 25.748^{Kg} in avoirdupois weight.

13. In 1 lb. 14 oz. 2.5 dr., how many grams?

14. In 3 lb. 2 oz. 11 dwt. 14.4 gr., how many grams?

15. Reduce 15^{T} to tons, etc., avoirdupois weight.

16. In 18.5184 grains how many milligrams?

17. What does a hektoliter of barley weigh?

1 gal. contains 10 lb

18. What is gained by buying 2000^{m} of cloth at \$1 per meter and selling it at \$1 per yard?

19. Express 1051^{g} in Troy weight.

20. Find the width of a cistern 2.5^{m} long and 1^{m} deep, which holds 3081.4 quarts.

EXAMPLE 19. Express £18 12s. 6d. in dollars.

£18 12s. 6d. = £18.625

Then since £1 = \$4.86⅔

∴ £18.625 = \$4.86⅔ × 18.625 = \$90.64.

EXAMPLE 20. Change \$453.21 to sterling money.

Since \$4.86⅔ = £1

$$\therefore \$1 = \frac{£1}{4.86\frac{2}{3}}$$

$$\therefore \$453.21 = \frac{£1 \times 453.21}{4.86\frac{2}{3}} = £93\ 2s.\ 6d.$$

EXAMPLE 21. Express 2000 piastres in francs.

2000 piastres = \$.044 × 2000 = \$88.

And since \$.193 = 1 franc

$$\therefore \$1 = \frac{1 \text{ franc}}{.193}$$

$$\$88 = \frac{1 \text{ franc} \times 88}{.193} = 455.96 \text{ francs.}$$

EXERCISE 20.

Express:

1. £27 6s. 8d. in dollars.
2. \$852.60 in sterling money.
3. \$119 in German money.
4. 756.60 marks in dollars.
5. 837½ francs in drachmas.
6. 95 pfennig in cents.
7. 8654.25 marks in dollars.
8. \$7500 in Austrian money.
9. £486 13s. 4d. in marks.

10. 3860 liras in Canadian money.
11. 3860 francs in United States money.
12. 3860 drachmas in dollars.
13. $386 in French, Belgian and Swiss money.
14. $1340 in Danish, Swedish and Norwegian money.
15. 25 cents in pfennig and in kreutzers.
16. $1 in rupees and annas.
17. £5 10s. in roubles and copecks.
18. 1000 francs in sterling money.
19. 1000 rupees in marks and pfennig.
20. 50 cents in ore, and in reis (Portugal).

LONGITUDE AND TIME.

The earth makes a complete revolution on its axis every 24 hours, although to our senses it appears that the sun makes a circuit of the earth. The 360° thus apparently traversed by the sun in 24 hours is just 15° for each hour, 15′ for each minute and 15″ for each second. From this it is evident that when the difference in longitude of two places is known the difference in time can be calculated, and *vice versa*.

Longitude is the number of degrees, etc., which a place is east or west of a given meridian.

In most countries longitude is reckoned from the meridian which passes through Greenwich, London.

Places distant from each other

15° of longitude differ 1 hour in time.
1° " " 4 minutes in time.
1′ " " 4 seconds "

EXAMPLE 22. What is the difference in time between two places whose difference in longitude is 48° 25′ 35″?

15 | 48° 25′ 35″

3 hr. 13 min. 42⅓ sec.

Since the number of hours in a day is $\frac{1}{15}$ the number of degrees the sun travels in that time, therefore $\frac{1}{15}$ of the difference of longitude expressed in degrees, minutes, and seconds is equal to the difference of time in hours, etc.

EXAMPLE 23. The difference in time between London and New York is 4 hrs. 55 mins. $37\frac{2}{3}$ secs. What is their difference in longitude?

4 hr.	55 min.	$37\frac{2}{3}$ sec.
		15
73°	54′	25″

Since there are 15 times as many degrees, minutes and seconds of longitude as there are hours, minutes and seconds of time, we simply multiply the hours, etc., by 15 and write it down degrees, etc.

EXERCISE 21.

1. Halifax is 63° 35′ 30″ west longitude. When it is noon at Greenwich what is the true time in Halifax?

2. Toronto is 79° 21′ west longitude. What is the difference in the true time of Toronto and Halifax?

3. St. John is 66° 3′ 30″ west longitude, Boston 71° 3′ 30″ and St. John's 52° 43′. What is the difference in true time between St. John and each of the other cities?

4. What is the difference in time between Charlottetown, which is 63° 7′ west longitude and Chicago which is 87° 38′ west.

5. The difference in the true time of two places is 1 hr. 22 min. 20 sec. What is the difference in longitude?

6. Two persons observed a particular star to be hidden by the moon, one seeing it at $21\frac{1}{2}$ o'clock, and the other at $23\frac{1}{4}$. What was the approximate difference in their longitude?

7. The standard time adopted in Nova Scotia is the true time of the 60th meridian. Is it faster or slower than Halifax true time, and how much?

8. A gentleman whose watch was set to Halifax true time arrived in Montreal and was informed that his watch was 39 min. 2 sec. faster than Montreal local or true time. What is the longitude of Montreal?

9. St. Petersburg is 30° 20′ east longitude. When it is 10 o'clock at St. John (66° 3′ 30″ west), what is the time in St. Petersburg?

10. If, on the occasion of a Dominion election, the polls open at 8 o'clock and close at 17, true time, how

much sooner will they open and close at Sydney, C.B., 60° 12′ 9″ west longitude, than at Vancouver, B.C., 123° 22′ 24″ west longitude?

11. If a telegraph message is sent at noon without any loss of time from London to Washington 77° 1′, at what time (true time) is it received?

12. What is the difference in time between Cape of Good Hope (18° 29′ E.) and Quebec 71° 13′ 45′ ?

13. A man travelling along the equator found when he stopped that his watch was 1 hr. 35 min. slow. Did he travel east or west and how many miles?

14. What is the difference in time of two places whose difference of longitude is 35° 12′ 15″?

15. Two places are 48° 24′ 36″, longitude, apart. What is the difference in their true time?

16. The difference in the true time of two cities is 2 hr. 15′ 27″. What number of degrees, etc., is one farther east than the other?

EXAMINATION PAPER No. 7.

1. What o'clock is it when the time from noon is $\frac{9}{11}$ of the time to midnight?

2. A man walks a certain distance and then rides back in 3 hrs., 25 mins. He could ride both ways in $2\frac{1}{2}$ hours. How long would it take him to walk both ways?

3. Reduce the difference between £.427083 and .23$\dot{4}\dot{5}$ of £6 17s. 6d. to the decimal of £5.

4. A river 5^{m} deep and 96^{m} wide flows 3.6^{Km} per hour. What weight of water does it carry to the sea in 20 minutes?

5. What is the value of a bar of aluminum 6^{m} long, 1^{dm} wide and 5^{cm} thick at 65 cents per pound?

THERMOMETERS.

There are two thermometers in common use,—the Fahrenheit and the Centigrade. The latter, only, is used for scientific purposes.

In preparing his thermometer, about 1720, Fahrenheit, of Amsterdam, by a mixture of ice and common salt, produced what he thought to be the greatest cold that could be produced by mechanical means and this he called *zero*. He divided the difference in temperature between this point and the point at which water boils into 212 equal parts *degrees*. By this scale he found that water freezes at 32 above zero.

In the Centigrade thermometer, first introduced by Celsius, of Sweden, about 1742, the freezing point of water is taken as 0°, or zero, and the boiling point as 100°.

Thus, the difference between the freezing and boiling points of water as indicated by the Fahrenheit thermometer is the difference between 212° and 32°, or 180°, while the same difference as indicated by the Centigrade scale is 100°; hence 100° Centigrade are equal to 180° Fahrenheit, or 5° C. = 9° F.

Temperature *below* zero on either thermometer is indicated by the minus sign. Thus 40° below zero is -- 40° C, or - 40° F.

It is believed that 273 C. is the point which marks the absence of all heat.

EXAMPLE 24. How is 100° F. indicated by a C. thermometer?

$100° - 32° = 68°$.

$68° \times \frac{5}{9} = 37\frac{7}{9}°$ C. Ans.

Since in the F. thermometer the freezing point of water is 32° *above* zero, we must subtract 32° from 100° in order to ascertain the number of degrees 100° F. is above the freezing point of water. Then since 100° C. = 180° F. each of the 68° F. = $\frac{5}{9}$ C. Therefore, 68° F. = $68° \times \frac{5}{9}$ or $37\frac{7}{9}$° C.

EXAMPLE 25. How would a temperature of 75° C. read on a Fahrenheit thermometer?

$75°\text{C.} \times \frac{9}{5} = 135°$ F.
$135° + 32° = 167°$ F.

Since 1° C $= \frac{9}{5}$° F., 75° C. = 135° F. But as this is 135° F. *above the freezing point of water*, we must add 32° to indicate the number of degrees *above zero.*

EXAMPLE 26. What is – 56°C. on the Fahrenheit scale?

$56°$ C. $\times \frac{9}{5} = 100.8°$ F.
$100.8° - 32° = 68.8°$ F.
As this is *below* zero, it is marked – 68.8° F.

56° C. = 100.8° F. But as the 56° C. is below the freezing point of water, the 100.8° F. is below the same point. And as zero or 0° F. is 32° below the freezing point of water, the difference between 100.8° and 32° is the distance below zero on the F. scale.

EXAMPLE 27. What is 12° F. on the C. thermometer?

32° F. – 12° F. = 20° F. = number of degrees below freezing point of water.

20° F. $\times \frac{5}{9} = 11\frac{1}{9}°$ C. = number of degrees below freezing point of water.

Hence, 12° F. $= -11\frac{1}{9}°$ on a Centigrade thermometer.

All exercises of converting readings of one scale into readings of the other can be done equally well, perhaps with greater facility, by referring to the *boiling point of water* instead of to zero. Thus:—

EXAMPLE 28. How is 100° F. indicated on a C. scale?

As water boils at 212° F., 100° F. is 112° below the boiling point of water.

112° F. $\times \frac{5}{9} = 62\frac{2}{9}°$ C. This is $62\frac{2}{9}°$ C. below the boiling point of water.

As water boils at 100° C., $62\frac{2}{9}°$ below this point is $37\frac{7}{9}°$ C. Ans.

EXAMPLE 29. What is – 56° C. on a Fahrenheit thermometer?

– 56° is 156° C. below the boiling point of water.

156° C. $\times \frac{9}{5} = 280.8°$ F. below the boiling point of water.

And as water boils at 212° F. *above* zero, 280.8° F. below that point must be 280.8° – 212° or 68.8° F. below zero, or – 68.8° F.

EXERCISE 22.

1. How would 65° C. read on a Fahrenheit thermometer?

2. What temperature Centigrade does 86° F. represent?

3. The temperature of a room is 20° C. What would a Fahrenheit thermometer indicate?

4. Alcohol boils at 173° F. What is this on a Centigrade scale?

5. Tin melts at 230° F. and lead at 334° F. At what degree Centigrade do they melt?

6. How would temperatures of 40°, 50°, 60°, 70°, 80°, 90°, 100° Centigrade be indicated by a Fahrenheit thermometer?

7. Express 113°, 131°, 140°, 176°, 203° Fahrenheit in the Centigrade scale.

8. 75° C. is how many degrees hotter than 149° F.?

9. How does 25° C. read on a Fahrenheit thermometer?

10. What is 40° F. on a centigrade thermometer?

11. How do 46.4° F., 125.6° F., 145.4° F., and 195.8° F. read on a centigrade thermometer?

12. Mercury freezes at – 39° C. At what degree F. does it freeze?

13. Silver melts at about 1000° F., copper at about 1090° F., gold at about 1250° F., and wrought iron at about 1650° F. How are their respective melting points indicated on the C. scale?

14. On January 30th, 1898, in one town in Nova Scotia the mercury in a C. thermometer stood 25° below zero, and in another town the mercury in a F. thermometer stood 13° below. Which was the colder?

EXAMINATION PAPER No. 8.

1. How many paces each 2 ft. 5 in. in length will a man require to take in walking 50 miles.

2. What does a fruit-seller gain by selling at 2 kreutzers apiece 5 doz. oranges which had cost him 25 centimes a dozen?

3. A train 88 yards long takes 6 seconds to pass a man walking by the side of the track at the rate of $3\frac{1}{2}$ miles per hour. At what rate per hour is the train moving?

4. A 45-foot ladder placed between two buildings, with one end on the ground, reaches one of them at a height of 24 ft. and the other at a height of 28 ft. How far are the buildings apart?

5. Divide .777777777 by .63, and multiply the quotient by the product of .09 and 10.

PERCENTAGES.

(For Explanation see Part III., Examples 21-25).

EXERCISE 23.

1. What is the difference between $6\frac{1}{4}\%$ of \$4800 and $16\frac{2}{3}\%$ of \$1800?

2. A man who had land worth \$4500 sold 20% of it, then $33\frac{1}{3}\%$ of the remainder, and then $12\frac{1}{2}\%$ of what was left. What was the value of the land which he still owned?

3. In a certain mixture of tea there are $18\frac{3}{4}$ lb. of Oolong, which is $12\frac{1}{2}\%$ of the whole. How many pounds in the mixture?

4. What per cent. is gained by a druggist who sells 1 lb. ap. for what 1 lb. av. cost him?

5. What per cent. is made by selling cloth at the same price per yard as it cost per meter?

6. $\frac{2}{3}$ is what per cent. of $\frac{3}{4}$?

7. $\frac{2}{3}$ is what per cent. less than $\frac{3}{4}$?

8. $\frac{3}{4}$ is what per cent. greater than $\frac{2}{3}$?

9. A piece of cloth, on being sponged, shrank 10% in length and 10% in breadth. What per cent. did it shrink?

10. What per cent. of 30 is 20? What per cent. of a cube, each edge of which is 30 inches long, is a cube each edge of which is 20 inches long?

11. The female population of a town of 52000 is what per cent. of the whole, when the 5434 electors are 22% of the males?

12. Two women buy pears at a cent each. One sells them at 4 for 5 cents, and the other at 3 for 4 cents. What per cent. does the second gain more than the first?

13. If I sell $\frac{5}{6}$ of an article for what $\frac{2}{7}$ of it cost me, what is my gain per cent.?

14. What per cent. of what he grinds does a miller receive who takes as his toll 4 quarts out of each bushel? And the flour which the miller keeps is what per cent. of the flour which the farmer receives?

15. Which is better, and how much, on a purchase of $2340 worth of goods, a wholesale discount of 25% with a cash discount of 10%, or a single discount of $33\frac{1}{3}$%?

16. A druggist buys acid at 40 cents a gallon, and adds water so that when he sells it at 30 cents per gallon he gains 40%. What is the per cent. of water in each gallon?

17. *A* sold a horse which cost him $200 to *B* at a gain of 10%; *B* sold him to *C* at a loss of 10%; *C* sold him to *D* at a gain of 10%; and *D* sold him to *A* at a loss of 10%. If the horse is still worth $200 what did *A* gain?

18. A gentleman bought two cottages and paid $1500 for one, and $2250 for the other. He sold each for the same sum, gaining the same per cent. on one as he lost on the other. What did he gain or lose on the transaction?

19. A property was sold at $7\frac{1}{2}$% below cost. Had it been sold 10% higher it would have been at a gain of $56. What was the cost?

20. A wholesale merchant sold to a retail dealer at a profit of $22\frac{1}{2}$%. The retail merchant failed and compromised with his creditors at 55 cents on the dollar. What per cent. did the wholesale dealer lose?

TAXES. COMMISSION. INSURANCE.

(For Explanation see Part III., Examples 26-30).

EXERCISE 24.

1. What tax is paid by a man rated $\frac{5}{8}\%$ on property valued at $8654?

2. What is the amount, at $.84 per cent., of a farmer's tax whose real estate is estimated at $4860, and personal property at $1175?

3. What per cent. of the area of England is the area of Nova Scotia, if England is 242 miles square and Nova Scotia 147?

4. In 1897-8 property in Halifax was taxed, for all purposes, 1.63%. What were the taxes of a man whose real estate was valued at $7450, and personal property at $1340?

5. At what rate is property assessed when an estate valued at $7860 pays a tax of $98.25?

6. What does an agent receive on sales aggregating $1275, on a commission of $1\frac{3}{4}\%$?

7. An attorney collected 70% of a debt of $6400, and charged $6\frac{1}{2}\%$ commission. How much did the creditor lose?

8. A merchant sent his agent $1680, with instructions to invest in goods, deducting his own commission of $1\frac{1}{2}\%$. What is the amount of the commission?

9. A fruit-grower sent a commission merchant in St. John 400 bbl. of apples which were sold at an average price of $1.35 per barrel, and the proceeds, less the commission, invested in goods. If the commission for selling is 6% and for buying $2\frac{1}{2}\%$, what does the merchant make?

10. Sent a commission merchant $2472 to be invested in flour, his own commission being 3%. How many bbls. of flour, at $4.80 per bbl., can he purchase?

11. In the last question, after paying $8\frac{3}{5}$ cents per bbl. freight, what must be charged per bbl. for the flour so as to gain $8\frac{1}{2}\%$?

12. What will it cost to insure a house worth \$6000 for $\frac{3}{4}$ of its value at a premium of $1\frac{1}{2}\%$?

13. The premium on a building insured at $\frac{2}{3}$ of its value at $1\frac{3}{4}\%$ is \$105. What is the value of the building?

14. A factory valued at \$16000, and its machinery, which cost \$8400, are both insured. The building is insured at $\frac{4}{5}$ of its value at $1\frac{3}{8}\%$, and the machinery at $\frac{3}{5}$ of its value at $2\frac{1}{4}\%$ What is the annual premium paid?

15. What sum of money must I send my agent so that after deducting his commission of $2\frac{1}{2}\%$ he may send me goods worth \$4290?

16. For what sum must a building worth \$4290 be insured at $2\frac{1}{2}\%$ premium so as to cover both the value of the building and the cost of insurance?

17. A building which cost \$2000 was fraudulently insured at 3%, so that when it was burned the owner gained \$425. What was the amount of the policy?

18. For what sum must a schooner which was sold at auction for \$5214 be insured at 2% so that in case it is lost the purchaser will lose exactly \$1000?

19. A man whose property is assessed $\frac{3}{5}\%$ pays \$20.40. If his real estate is worth \$2500, what is the value of his personal property?

20. What per cent. profit is made by a merchant who buys at 20% and $7\frac{1}{2}$ (cash discount) from list prices, and sells at 10% and 3% from list prices?

CUSTOMS HOUSE BUSINESS.

The revenue necessary to defray the expenses of the Dominion Government is largely derived from taxes imposed upon goods imported from foreign countries. These taxes are known as **Customs Duties.**

Taxes on articles manufactured within the country are called **Excise Taxes.**

Duties are collected at seaport and other towns called **Ports of Entry.** These Ports of Entry are furnished with customs houses where government officers collect the duties.

Duties are either **Specific** or **Ad Valorem.**

Specific duties are assessed on imported goods with reference to their quantity or weight and not to their value.

Ad valorem duties are percentage taxes on the cost of goods in the country from which they are brought. Importers are required to submit to the customs officers an invoice of the goods, showing the cost of each article in the country where it was purchased.

EXERCISE 25.

1. What is the duty on a dozen books, weighing 18 lb., at 6 cents per lb.?

2. What is the duty, at 18%, on 12650 lb. of cordage, invoiced at 15 cents per lb.?

3. What is the duty on 12 dozen watches at $124 per doz. at 25%, and 20 jewel cases at $1.35 each at 30%?

4. What is the rate of duty when $94.50 is paid on goods invoiced at $1260?

5. What is the duty on 75 kegs of prunes of an average weight of 104 lb., at $1\frac{1}{4}$ cents per lb.; allowing a reduction of 7% for *tare*, that is for the weight of kegs, etc.?

6. What duty at 30% *ad valorem* must be paid at St. John on silk imported from England and invoiced at £872 2s. 6d.?

7. A merchant in Halifax made an importation of goods from the United States invoiced at $9865. On

goods invoiced at $2150 the duty was 17½%; on goods amounting to $3720 the duty was 22½%; goods costing $2612 were free of duty; and on the balance the duty was 30%. What was the whole amount of the duties?

8. When the duty on cut tobacco is 45 cents per lb. and 12½% *ad valorem*, what must be paid at the Charlottetown customs house on 50 casks, each weighing 112 lb., tare 5%, which cost in Virginia 15 cents per lb.?

9. At 30% *ad valorem* what is the duty—Canadian currency—on an importation of 100 doz. kid gloves invoiced at 75 francs per doz.?

10. What is the duty at 6 cents per lb. and 25% *ad valorem* on woollens from England weighing 840 lb., tare 8%, and invoiced at £756 17s. 6d.?

EXAMINATION PAPER No. 9.

1. What per cent. of the letters are vowels in the sentence, *I ought, therefore I can?*

2. A gentleman whose house was destroyed by fire received from the underwriters $2945, which was ⅘ of the amount of the policy. The insurance was ¾ of the value of the house and the premium paid was $66.24. What was the loss?

3. What direct discount is equal to a trade discount of 17½% and 8%?

4. A number is increased by 4% of itself, and the number obtained is increased by 18% of itself. If the last number is 767, what was the first?

5. The taxable property in a school section is $38600, and there are 18 persons who pay a poll tax of $2 each. When it is necessary to raise $540 for school purposes, what must a man pay whose real estate is valued at $920 and his personal property at $275?

INTEREST.

EXERCISE 26.

(For Explanations see Part III., Examples 31-36).

Find the interest of :

1. \$650 for 3 years at 6%.
2. \$875.60 for $2\frac{1}{2}$ years at 7%.
3. \$892.85 for $3\frac{1}{4}$ years at $5\frac{1}{2}$%.
4. \$1250 for 1 year 9 mos. at $6\frac{1}{4}$%.
5. \$342.50 from June 3, 1892, to Dec. 24, 1894, at 6%.
6. \$88.70 from Jan. 10, 1896, to Nov. 30, 1896, at 5%.
7. \$2968.80 for 2 years 155 days at 4%.
8. \$1234.50 from Oct. 3, 1897, to March 15, 1899, at 5%.

Find the compound interest of :

9. \$350 for 3 years at 6%.
10. \$426.75 for 4 years at 5%.
11. \$2445.60 for 3 years at 6%.
12. \$750 for $1\frac{1}{2}$ years at 6% interest payable half-yearly.
13. \$450 for 2 years at 5%, interest payable half-yearly.
14. \$200 for 1 year 9 mos. at 4%, interest payable quarterly.

Find the amount of :

15. \$194.60 for 5 years at $4\frac{1}{2}$%.
16. \$349 for 2 years at 6% compound interest.
17. \$4000 from Feb. 4, 1890, to Nov. 2, 1893, at 6%.
18. \$420 from May 16, 1890, to Jan. 7, 1893, at 6%, compound interest.
19. \$2000 from Sept. 18, 1893, to Feb. 11, 1896, at 5%, compound interest.
20. \$500 from May 1, 1898, to July 15, 1899, interest compounded quarterly at 8%.

Find the rate per cent. when :

21. \$387 amounts to \$468.27 in 3 years.
22. The interest of \$137.60 for $3\frac{1}{2}$ years is \$24.08.

23. The interest of $3500 for 11 months is $152.40.

In what time will :

24. $825 produce $67.81 interest, at 6% ?

25. $706.60 produce $80.37 interest, at $6\frac{1}{2}$% ?

26. $875.45 amount to $959.06, at 6% ?

What principal will give :

27. $1618.75 interest in $3\frac{1}{2}$ years at 5% ?

28. An amount of $1052.84 in 2 years 7 mos., at $5\frac{1}{4}$% ?

29. $23.10 interest, at $5\frac{1}{4}$, from April 1, 1894, to Jan. 18, 1895 ?

30. An amount of $538.63 from Sept. 11, 1895, to Dec. 25, 1896 at 6% ?

EXAMINATION PAPER No. 10.

1. A steamer from London to St. John traversed $10\frac{1}{2}$ degrees daily. What was the length of time from noon one day to noon the next?

2. An agent who collected a debt on a commission of $1\frac{3}{8}$%, sent his principal $1247.12. What was the amount of the debt?

3. From a lot of land 30 rods square were sold 300 square rods. What is the value of $53\frac{1}{3}$% of the remainder at $46 per acre?

4. A and B have respectively 7% more and 5% less money than C, and the three together have $1751.60. What sum has C ?

5. If a gallon of paint costing $1.50 will paint 150 square feet of board fence, what will the paint cost for a fence 5 ft. high, enclosing in four-sided form of least perimeter, a field containing 100 square rods ?

PARTIAL PAYMENTS.

EXAMPLE 30.

$4000. TORONTO, June 1, 1892.

Two years after date I promise to pay William Smith, or order, four thousand dollars, for value received, with interest at 7 per cent.

RICHARD POWELL.

On this note were the following endorsements :

Sept. 15, 1892, $450; Dec. 15, 1892, $50; March 1, 1893, $500; Jan. 1, 1894, $1000.

What was payable when note became due on June 4, '94?

Interest on $4000 from June 1, '92, to Sept. 15, '92,—106 days = - - - - - -	$81.31
Interest on $3550 from Sept. 15, '92, to Dec. 15, '92,—91 days = - - - - -	61.95
Interest on $3500 from Dec. 15, '92, to Mar. 1, '93,—76 days = - - - - - -	51.01
Interest on $3000 from Mar. 1, '93, to Jan. 1, '94,—306 days = - - - - -	176.06
Interest on $2000 from Jan. 1, '94, to June 4, '94,—154 days = - - - - - -	59.07
Amount of interest - - - -	$429.40

Balance due = $2000 + $429.40 = $2429.40.

NOTE.—During the first 106 days Richard Powell owes $4000 and pays interest accordingly; during the next 91 days he owes and pays interest on $3550, and so on to June 4, '94, he continues to pay interest on the amount of his indebtedness. The whole interest with the balance of principal unpaid amounts to $2429.40.

As the note anticipates and the law allows only simple interest, it is obviously unjust to compound the interest every time a payment is made. Such a method would operate against the creditor, in this example, to the extent of $26.10.

The method may be stated as follows :

Find the interest of the principal to the time of the first payment; the interest of the balance to the time of the next payment, and so on to the time of settlement. The balance of the principal unpaid, together with the several amounts of interest computed, is the amount of the debt.

This method is known as *Merchants' Rule*. It is used by all bankers and is legal in Canada. It is sometimes expressed and applied as follows:

Find the amount of the principal for the entire time; find the amount of each payment from the time that it was made to the time of settlement; and from the first amount subtract the sum of the amounts of the several payments.

EXERCISE 26a.

$400. ST. JOHN, Jan. 1, 1891.

1. One year after date, for value received, I promise to pay Donald Elsdon, or order, four hundred dollars with interest at 7%. JACKSON GRANT.

Indorsements:—March 16, 1891, $200; July 1, 1891, $100. What was due at maturity?

2. Find the balance due April 15, 1896, on a note of $250.60, given July 7, 1895, interest at 7%, on which the following payments have been made:—Sept. 20, 1895, $80; Jan. 1, 1896, $50; March 13, 1896, $50.

3. A note of $500 was given Jan. 1, 1896.

Indorsements:—Jan. 20, '96, $100; Feb. 10, '96, $50; Feb. 25, '96, $100; March 1, 1896, $150. What was due April 4, 1896, interest 6%?

4. A note of $900, dated Sept. 1, 1895, with the following indorsements:—Oct. 18, '95, $150; Dec. 22, '95, $200; March 15, '96, $300. Interest being at 7%, what was due on the note July 19, 1896?

5. The sum of $725.25 was loaned on March 15, 1896, at 6%. Payments were made as follows:—April 3, $170; May 20, $245.30; June 17, $87.50. How much was due Sept. 5, 1896?

BANK DISCOUNT.

(For Explanations see Part III., Examples 37-38).

EXERCISE 27.

Find the bank discount and proceeds of the following:

	FACE.	DATE.	TIME.	WHEN DISCOUNTED.	RATE.
1.	$850	Feb. 5, '91.	2 mo.	Feb. 23, '91.	6 %.
2.	$750	Sept. 4, '96.	3 mo.	Oct. 2, '96.	4 %.
3.	$500	Mar. 17, '95.	90 da.	May 22, '95.	5 %.
4.	$75	July 8, '96.	90 da.	Aug. 4, '96.	$5\frac{1}{2}$ %.
5.	$1260	May 14, '97.	60 da.	June 25, '96.	7 %.
6.	$5430	Feb. 10, '95.	3 mo.	April 9, '95.	6 %.
7.	$5430	Feb. 10, '95.	90 da.	April 9, '95.	6 %.
8.	$485	Nov. 28, '97.	90 da.	Jan. 3, '98.	5 %.
9.	$485	Nov. 28, '97.	3 mo.	Jan. 3, '98.	5 %.
10.	$485	Nov. 30, '97.	3 mo.	Jan. 3, '98.	5 %.

TRUE DISCOUNT AND PRESENT WORTH.

$106. TRURO, June 10, '97.

Twelve months after date, for value received, I promise to pay James Smith, or order, the sum of one hundred six dollars, without interest. JOHN BROWN.

What will this note be worth to James Smith on June 10, '98? Just $106, for on that day John Brown will pay him that amount. But what is the value of the note on June 10, '97,—the day on which it was given? It is worth such a sum as, being put at interest, will amount to $106 on June 10, '98. If money earns 6%, what sum will amount to $106 in 1 year? $100. What then is the present worth of $106 due a year hence when money is worth 6%? $100. And what is the true discount, or the sum which would be deducted from such a debt, if it were paid twelve months before it was due? $6.

It is evident then that the **Present Worth** of any sum of money due at some future time is the sum which, put at interest for the given time and rate, will amount to the given debt.

And **True Discount** is the difference between a debt and its present worth ; or the interest on the present worth for the given time.

EXAMPLE 31. Find the Present Worth and True Discount of $942.57, due 2 years hence, interest 4%.

Amt. of $100 for 2 years at 4% = $108
Hence the Present Worth of $108 = $100
And " " " $1 = $\frac{\$100}{108}$

$\therefore$ " " " $942.57 = $\frac{\$100 \times 942.57}{108}$ = $872.75

True Discount = $942.57 − $872.75 = $69.82.

EXAMPLE 32. A debt of $665.60 contracted on Feb. 11, to be paid in 6 months, without interest, was paid on March 18. Find the Discount, money being worth 6%.

The debt becomes due on Aug. 11.
Time from March 18 to Aug. 11. = 146 days.
Amt. of $100 for 146 days at 6% = $102.40
Hence the True Discount of $102.40 = $2.40
And " " $1 = $\frac{\$2.40}{102.40}$

$\therefore$ " " $665.60 = $\frac{\$2.40 \times 665.60}{102.40}$ = $15.60

Present Worth = $665.60 − $15.60 = $650.

EXERCISE 28.

Find the Present Worth and True Discount of :

1. $825 due in 2 years at 5%.
2. $1062 due in 3 years at 6%.
3. $352.30 due in 3 years at 4%.
4. $1518.55 due in 3 years at 7%.
5. $10868.75 due in $3\frac{1}{2}$ years at 5%.
6. $90.88 due in $4\frac{1}{2}$ years at $5\frac{1}{2}$%.

7. A piece of land purchased for \$999 on July 1, '93, to be paid at the end of 2 years, but paid on March 1, '94, money being worth 6%.

8. \$10049.31 due on June 3, but paid on May 4, money worth 6%.

9. \$578.80 due in 8 years at 5%, when money can be loaned at 5% compound interest.

(Here the amt. of \$100 for given time is \$115.76).

10. \$764.28 due in $1\frac{1}{2}$ years, when money can be loaned at 6%, interest compounded half-yearly.

11. What is the difference between the true and bank discount of \$2000 at 5% for 4 months, days of grace not reckoned.

12. Whether is it cheaper to buy a horse for \$210 cash or for \$220 on a credit of 10 months, money worth 6%.

13. I am offered a house for \$2838, to be paid at the end of 18 months. If money can be hired at 5%, what is that equivalent to in cash?

14. The interest of any sum of money for 1 year at 5%, is $\frac{5}{100}$ of that sum, and the true discount is $\frac{5}{105}$. On what sum is the interest \$4 greater than the discount?

15. On what sum does the true discount for 6 mos. at 6% amount to \$24?

EXAMINATION PAPER No. 11.

1. In what time will a sum of money double itself at $6\frac{1}{2}$%, simple interest?

2. What sum will amount to \$635 in $4\frac{1}{2}$ years at 6%, simple interest?

3. If you place a 3-month note for \$100 with a bank when money is worth 8%, what sum do you receive, without allowing for days of grace? What fraction of the money you receive for use do you pay to the bank? What fraction would you require to pay in the case of a private loan at the same rate?

4. A merchant who used as his private mark the word "*Precaution*," marked a web of silk, "Cost price—p. ta., "Selling price—r. ea." After giving a customer a reduction of 25 cents per yard, what per cent. of profit does he make?

5. At an examination there were 7 candidates at the age of 13, 4 at 14, 11 at 15, 5 at 16 and 9 at 17. What was the average age?

STOCKS. BONDS. BROKERAGE.

When two or more individuals enter into an agreement for the purpose of carrying on a commercial or other enterprise, the association is called a **Partnership,** and each individual a **Partner.**

If, however, they secure a charter from the government, defining the objects, powers and limitations of the Company, and organize by the election of such officers as a president, secretary, treasurer and board of directors, the association is called a **Joint Stock Company.**

"One of the special advantages of a charter is that it commonly limits each stockholder's liability to the amount of the face value of his shares, whereas in an unincorporated company or firm, each member is liable for all debts of the company."

The money required for carrying on the enterprise is divided into shares of a definite value. Each shareholder receives a *certificate* showing the number of shares he is entitled to, their par value, and the amount he has paid on them.

The value of the shares of stock named in the certificate is called **Par Value.**

When the business of the Company is very profitable, the market value of the stock is *high* or **above par,** or **at a premium;** when the business is unprofitable the shares are *low* or **below par,** or **at a discount.**

When a $100-share sells at $100 it is at *par;* when it sells for $112 it is at 12% *premium;* and when it sells for $96 it is at a *discount* of 4%.

The balance of the gross earnings of a Company, **left after paying expenses, is called Net Profit.**

Profit distributed among the shareholders is called **Dividend.**

Dividends are declared annually, semi-annually or quarterly.

Dividends and Assessments are always expressed as percentages of the par value.

When the Government of the Dominion, or of any province, or the municipal government of any county, city or town, or when the directors of any incorporated company wish to raise funds, *bonds* are prepared and sold. The bonds are secured by the property of those who issue them, and bear a fixed rate of interest payable annually, semi-annually or quarterly.

A **Bond** is a written obligation under seal securing the payment of a sum of money at or before a specified time.

The bonds of a Company are secured by a mortgage on its assets. The most common form of bonds are government bonds, railroad bonds, city bonds, etc.

Government bonds are of various kinds, and they are briefly described by abbreviations for rate of interest, date of payment, etc.

Thus: U. S. 4's 1906, reg., means United States registered bonds, bearing 4% interest, payable in 1906. Dominion 3's, 5-20, '87, means Dominion government bonds, bearing interest at 3%, issued in 1887, and payable any time between 5 and 20 years which the government chooses. British *Consols*, "consolidated annuities," are perpetual annuities bearing interest at 3%.

Bonds are sometimes called **Stock,** because both are bought and sold in the same way. But Bonds pay a regular interest at a fixed rate, whereas the income from Stocks, depending on the profits made from time to time, is variable.

A person whose business is the purchase and sale of Stocks and Bonds, is a **Broker.** His commission is called **Brokerage,** and is always reckoned upon the par value of the stocks or bonds.

How does a broker's commission differ from that of a commission merchant?

EXAMPLE 35. What is the cost of 50 rail-road shares at 94, brokerage $1\frac{1}{2}\%$?

Cost of 1 share $= \$94 + \$1\frac{1}{2} = \$95.50$

" " 60 shares $= \$95.50 \times 60 = \$573.$

EXAMPLE 36. How much stock at a premium of 8%, brokerage $\frac{3}{8}$%, can be purchased for $1734?

Cost of $100 stock = $108 + $\frac{3}{8}$ = 108\frac{3}{8}$.

Since 108\frac{3}{8}$ will buy $100 stock

$$\therefore \quad \$1 \quad " \quad " \quad \frac{\$100}{108\frac{3}{8}} \quad "$$

$$\therefore \quad \$1734 \quad " \quad " \quad \frac{\$100 \times 1734}{108\frac{3}{8}} = \$1600.$$

EXAMPLE 37. What sum must be invested in Dominion bonds, 5's at 108, to secure an annual income of $1500?

Since $5 is received by investing $108

$$\therefore \quad \$1 \quad " \quad " \quad \frac{\$108}{5}$$

$$\therefore \quad \$1500 \quad " \quad " \quad \frac{\$108 \times 1500}{5} = \$32400.$$

EXAMPLE 38. What is the income from an investment of $1898 in $3\frac{1}{2}$% stock at $87\frac{1}{4}$, brokerage $\frac{1}{2}$%.

Since 87\frac{3}{4}$ buys $100 stock

$$\therefore \quad \$1 \quad " \quad \frac{\$100}{87\frac{3}{4}} \quad "$$

$$\therefore \quad \$1898 \quad " \quad \frac{\$100 \times 1898}{87\frac{3}{4}} = 2162.93+.$$

Then $3\frac{1}{2}$% of $2162.93 = $75.60 income.

EXAMPLE 39. At what rate must 6% bonds be purchased to yield annually 5% of the investment?

Since it is desired to realize 5% of the investment, and the stock yields an income of $6 per share, therefore $6 must be 5% of the price of a share. Thus:

Since 5% of price of share = $6

$$\therefore \quad 1\% \quad " \quad " \quad = \frac{\$6}{5}$$

$$\therefore \quad 100\% \quad " \quad " \quad = \frac{\$6 \times 100}{100} = \$120.$$

EXERCISE 29.

1. What is the cost of 50 shares Nicaragua Canal stock at $50\frac{1}{2}$, brokerage $\frac{1}{8}\%$?

2. Find the value of 150 shares sugar refinery stock at $12\frac{1}{2}\%$ premium, brokerage $\frac{1}{4}\%$.

3. What must be paid for 220 shares electric railway stock at $94\frac{1}{8}$, brokerage $\frac{3}{8}$?

4. How much stock at 95, brokerage $\frac{1}{2}$, can be bought for $955?

5. What amount of Suez Canal stock at $181\frac{1}{4}$, brokerage $1\frac{1}{2}$, can be bought for $7310?

6. How many shares C.P.R. stock $107\frac{3}{4}$, brokerage $\frac{1}{4}$, can be bought for $5400?

7. What annual income is derived from $22500 invested in Dominion 5's at 105?

8. A broker sold 600 shares Portland Gas Co. stock at 61, and invested in City of St. John bonds at 109. Find amount of brokerage, the rate for selling being $\frac{3}{4}\%$, and for buying $\frac{3}{8}\%$.

9. What is the yearly income when $18350 is invested in $6\frac{1}{4}\%$ bonds at 127, brokerage $1\frac{1}{2}$?

10. What is gained by investing $10000 in British Consols at $103\frac{1}{2}$, and selling immediately at 105, brokerage in each transaction $\frac{3}{4}\%$?

11. Which is the better investment $4\frac{1}{2}\%$ bonds at 92, brokerage $1\frac{1}{8}$, or $5\frac{1}{2}$ bonds at 106, brokerage $1\frac{3}{4}$?

Which is the larger fraction $\frac{4\frac{1}{2}}{93\frac{1}{8}}$ or $\frac{5\frac{1}{2}}{107\frac{3}{4}}$?

12. Which will yield the better income, 8% bonds at 124, brokerage $2\frac{1}{4}\%$, or $6\frac{1}{2}$ bonds at 110, brokerage $1\frac{7}{8}\%$?

13. What must I pay for 5% stock that the investment may yield 8%?

14. At what rate must I buy Midland Railway stock, paying 6%, to receive an income of $7\frac{1}{2}\%$?

15. A man who bought bank stock at $108\frac{1}{2}$ received $275 when a 5% dividend was declared by the bank. How much money did he invest?

16. In the United States when gold is quoted at $102\frac{2}{3}$, what is the currency value of $3700 of gold?

17. What sum must be invested in U. S. 4's at $121\frac{1}{4}$, brokerage at $\frac{1}{8}$%, to secure an annual income of $900?

18. How much must I pay for Bank of Montreal stock which pays a dividend of 11%, so that I may make 5% on my investment?

19. Which investment will give me the better income, $5\frac{1}{2}$% stocks at 96, or $6\frac{1}{2}$% stocks at 106?

20. A man sells $3\frac{1}{2}$% stocks at 92, and invests in $4\frac{1}{2}$% stocks at par. By what fraction is his income increased?

21. By investing in Halifax city 5% bonds I made $6\frac{1}{2}$%. At what rate did I buy the bonds?

22. A gas company declares a dividend of $15\frac{1}{2}$%. What will a man receive who owns 28 $50-shares?

23. To secure an annual revenue of $900, what capital must be invested in 5% bonds at 80?

24. A man bought 120 shares of sugar refinery stock at 44, and after holding it a year sold at a premium of 4%. What did he gain?

25. A cotton factory company, whose capital stock was $30000, declared a semi-annual dividend of $4\frac{1}{2}$%, and passed $864.50 to a reserve fund. What were the net earnings?

EXAMINATION PAPER No. 12.

1. How many gallons of water must be added to 80 gallons of acid 75% pure, so that it may be only 60% pure?

2. When money is worth 5%, how much more money can you hire for a year from a private individual for $50, than from a bank?

3. When wheat is selling at $1 per bushel, what does a man gain or lose who buys 30000 bu. by the Canadian bushel and sells at the same rate by the United States bushel?

4. A building worth $3000 was insured for $\frac{5}{6}$ of its value at $2\frac{1}{2}$%. The company, not wishing to carry the entire

risk, reinsured $\frac{4}{5}$ of the amount of the policy with another company at $2\frac{1}{4}\%$. What amount of the premium paid by the owner does the first company retain, and what will be the company's actual loss if the building is destroyed before the end of one year?

5. When 35 lb. of tea which cost 21 cents per lb. is mixed with 29 lb. which cost 30 cents, what must be the selling price so that a profit of 20% may be realized?

EXCHANGE.

Exchange is a method of making payments in distant places without transmitting money.

The business is carried on chiefly by banks which charge a small per centage for each transaction.

A **Draft** or **Bill of Exchange** is a written request or order upon one person or bank to pay a certain sum to another person or to his order, at a specified time.

The **Drawer** or **Maker** is the person or bank whose name is signed to the draft.

The **Drawee** is the person or bank to whom the order is addressed and on whom it is drawn.

The **Payee** is the person to whom the money is to be paid.

The **Buyer** or **Remitter** is the person who purchases the bill. In some cases he is himself the payee.

It is generally safer and more convenient for persons going abroad to take with them one or more bills of exchange on foreign banks, payable to themselves, than to carry large sums in bank notes or gold.

When a Drawee accepts a draft he writes the word "accepted," with his name, and the date, across the face of the draft. He thus assumes the obligation of paying it at maturity.

When the Payee writes his name on the back of a bill or draft he is said to *endorse* the bill, and it is then payable to the bearer. The payee may, however, endorse it to some particular individual, who in turn may endorse it either in blank or to some other person. Endorsers become separately responsible for the amount of the bill in case the

drawee fails to meet the payment. A draft made payable to *bearer* is of course transferable without endorsement.

A **Demand Draft** is a draft payable on presentation.

In an "**At Sight**" draft three days of grace are allowed. In other respects it is the same as a Demand draft.

A **Time Draft** is a draft payable at a specified time after date, or after sight.

In Canada three days of grace are allowed on all other than Demand drafts.

DOMESTIC OR INLAND EXCHANGE.

Domestic or Inland Exchange treats of remittances made from one place to another in the same country.

Domestic exchange is generally at par, and the cost of a draft above its face value is usually only a small charge made by the bank for its trouble. But it sometimes happens, for instance, that Vancouver banks have not sufficient money on deposit in Halifax to meet the drafts they are making upon Halifax. As it may then be necessary to go to the trouble and expense of forwarding money to Halifax by express, drafts on Halifax are likely to be for the time **at a premium**. Or, again, if large sums paid in at Winnipeg are drawn at St. John ; in other words, if many drafts on St. John are purchased at Winnipeg, the St. John bankers will have surplus funds at Winnipeg on which they are receiving no interest. To save the expense of bringing the money by express from Winnipeg they may sell drafts on Winnipeg *at a discount.*

The *premium* or *discount* will never be greater than the cost of sending the money by express from the one place to the other.

This premium or discount is called the **course of exchange** and depends largely on the **balance of trade.** If Canada sells to Britain more than Britain sells to Canada, then British merchants will be making payments to Canadian merchants in money. There will be more Canadian drafts purchased in Britain than British drafts purchased in Canada. So the British merchants will find that exchange with Canada is *at a premium.* The same may happen between Winnipeg and Halifax when Manitoba sells to Nova Scotia more wheat than is equivalent to the quantity of fish, apples or other Nova Scotian export which they buy.

FORM OF DRAFT.

$372.45. ST. JOHN, July 20, 1897.

At sight, pay to the order of Lawrence, Thorne & Co., three hundred seventy-two $\frac{45}{100}$ dollars, value received, and charge to the account of

HARVEY ELLIOTT.

To JONES & Co., Bankers,
Toronto.

EXAMPLE 40. What will a sight draft on Quebec for $1200 cost, at $\frac{1}{8}\%$ premium?

$1200 + $\frac{1}{8}\%$ of $1200 = $1201.50.

EXAMPLE 41.

$580. CHARLOTTETOWN, June 12, 1897.

Thirty days after sight, pay to G. U. Hay or bearer, five hundred eighty dollars, value received, and charge the same to the account of

J. D. SEAMAN.

To HARRIS & WOOD, Bankers,
St John.

What is the cost of the above draft, exchange being $\frac{1}{4}\%$ premium and money 6%?

$580 + $\frac{1}{4}\%$ of $580 = $581.45.

$581.45 − $3.15, bank discount on $580 for 33 days = $578.30.

If this draft were payable at sight its cost would be $581.45. But as the banker in St. John will have the use of the $580 for 33 days before he pays it, he allows bank discount on the face of the draft for that time. Hence the draft will cost $581.45 less $3.15, or $578.30.

EXERCISE 30.

What is the cost of a draft for:

1. $850 on Montreal at $\frac{1}{10}\%$ premium?
2. $1560 on New Westminster at $\frac{1}{4}\%$ discount?
3. $872.50 on Toronto at $\frac{1}{8}\%$ premium?
4. $210.40 on Fredericton at $\frac{1}{4}\%$ premium?

5. \$4860 on Winnipeg at $\frac{1}{8}\%$ discount?

6. \$640 on San Francisco at $\frac{3}{8}\%$ premium?

7. What is the cost of a 60-day draft on Boston for \$3560, exchange $\frac{1}{2}\%$ premium and money 6%?

8. What must be paid for a 10-day draft for \$1000, exchange $\frac{1}{10}\%$ discount, money 5%?

9. What must be paid for a draft of \$750, at 30 days, exchange being at $\frac{5}{8}\%$ premium and interest at 6%?

10. How large a sight draft on St. Louis, at $1\frac{1}{2}\%$ premium can be purchased for \$5000?

11. What is the value of a sight draft on Ottawa which cost \$2000, exchange $\frac{3}{8}\%$ premium?

12. A commission merchant sold a consignment of fruit in Halifax for \$960, and after deducting his commission of 4%, he purchased a sight draft on New York at $\frac{1}{8}\%$ premium. What was the face of the draft?

FOREIGN EXCHANGE.

Foreign Exchange deals with drafts drawn in one country and payable in another.

Foreign bills are generally, though not always, issued in sets of three, called respectively the *first*, *second* and *third* of exchange. They are transmitted by different routes, or on different days in order to minimize the danger of accident or miscarriage. The one reaching its destination first is paid at sight or after the time specified, and the others become void.

FORM.

Exchange for
£425. HALIFAX, Aug. 20, 1898.

At sight of this first of exchange (second and third of same date and tenor unpaid), pay to Murray, Fyshe & Co., four hundred twenty-five pounds and charge the same, as per advice, to

WHITEWAY BROS.

BAKER & SONS, Bankers,
London.

The Parliament of Canada fixed the *par value* of a pound sterling at $\$\frac{40}{9}$ or $\$4.44\frac{4}{9}$. But the *intrinsic* value of a pound (that is, the actual value of the gold in one pound sterling) is $\$4.86\frac{2}{3}$, or $9\frac{1}{2}\%$ above the old par value. Hence exchange between Canada or the United States and Great Britain is really at commercial par when it is quoted at a premium of $9\frac{1}{2}\%$.

Long before the introduction of dollars and cents the American Colonies of Great Britain quoted the value of £90 Sterling as equal to £100 Colonial money. This is known as the *old par value*. But the £90 Sterling eventually became worth, in the most of the Colonies, £109½, which, is $9\frac{1}{2}$ per cent. premium on the old par value. In Nova Scotia, before Confederation, a new par value was fixed at $12\frac{1}{2}\%$ premium, which meant that £90 Sterling was equal to £112½ Nova Scotian. The Nova Scotia pound was then changed into dollars at the ratio of $4 equal to £1. The Sterling pound was then equal to $5.

At Confederation the new par value was fixed at 9½% premium on the old, which made £90 Sterling equal to £109½ Canadian, equal to, on the 'change to dollars and cents, $4.38 Canadian, since $4 was reckoned as equivalent to £1 Canadian.

The force of habit is illustrated by the quotation of Sterling exchange on the old par value of £100 Colonial = £90 Sterling, or £90 Sterling = $400 Canadian, or £1 = $\$\frac{40}{9}$. By adding $9\frac{1}{2}\%$ to $\$\frac{40}{9}$ we get $\$4.86\frac{2}{3}$, or the present value of £1 Sterling.

In the following examples and in Exercise 20, we assume that a system of exchange, on the basis of the values given on page 41, exists between Canada and each of the countries named.

EXAMPLE 42. What must be paid in Canada for an English draft of £750, exchange $10\frac{1}{4}$ premium?

$$\text{Price of £1} = \frac{\$40}{9} \times \frac{110\frac{1}{4}}{100}$$

$$\text{“ “ £750} = \frac{\$40}{9} \times \frac{100\frac{1}{4}}{100} \times \frac{750}{1} = \$3675.$$

EXAMPLE 43. How large a bill of exchange on Edin-

burgh can be bought for $3200, when sterling exchange is quoted at $4.88½?

Since $4.88½ will buy £1

$1 " " $\frac{£1}{4.885}$

$3200 " " $\frac{£1 \times 3200}{4.885} = £655$ 1s. 3¾d.

EXAMPLE 44. What must be paid for a draft on Paris for 5000 francs, exchange being 3½% premium?

Price of 1 franc at par = $.193

" " 1 " " 3½ premium = $.193 $\times \frac{103\frac{1}{2}}{100}$

" " 5000 " " " " = $\frac{\$.193 \times 207 \times 5000}{200}$ =

$998.78.

EXAMPLE 45. What will a draft on Marseilles for 5000 francs cost, when a franc is worth $.198.

Cost of 1 franc = $.198

" " 5000 " = $.198 × 5000 = $990.

EXAMPLE 46. Find the cost of a draft on Berlin for 4000 marks, exchange 1¼% discount.

Cost of 1 mark at par = $.238

" " 1 " " 1¼ discount = $.238 $\times \frac{98\frac{3}{4}}{100}$

" " 4000 " " " " = $\frac{\$.238 \times 395 \times 400}{400}$ = $940.10

EXAMPLE 47. Find the face of a draft on Antwerp which can be purchased for $362.75 when the mark is quoted at $.243.

Face of draft purchased for $.243 = 1 mark.

" " " " " $1 = $\frac{1 \text{ mark}}{.243}$

" " " " " $362.75 = $\frac{1 \text{ mark} \times 362.75}{.243}$

= 1492.80 marks.

EXAMPLE 48. What will it cost to send 2000 roubles to St. Petersburgh when the rouble is quoted at $.562?

Cost of 1 rouble = $.562.
" " 2000 roubles = $.562 × 2000 = $1124.

EXERCISE 31.

1. What must be paid in Halifax for a bill of exchange on London for £354 10s., when sterling exchange is quoted at 4.87\frac{1}{4}$?

2. What is the cost of a bill of exchange on Dublin when sterling exchange is 8$\frac{1}{2}$% premium?

3. What is the cost in St. John of a sight draft on London for £312 15s 5d., when exchange is $4.87?

4. How much must be paid for a bill of exchange on Paris for 3500 francs, when 1 franc costs $.195?

5. What sum will purchase a 7000-franc bill of exchange on Paris at 5% premium?

6. What will a sterling bill for £317 9s. cost in Charlottetown when exchange is 4.90\frac{1}{2}$?

7. Find the cost of a bill of exchange on Geneva for 7250 francs at a premium of 10%.

8. A gentleman visiting Europe wishes to carry with him a bill of exchange on a bank in Naples for 1000 lira. What will it cost him in Halifax, exchange being 6% premium?

9. Find the price of a sight draft on Hamburg for 8540 marks, when the mark is quoted at $.234.

10. How many dollars are equal to £1 sterling, when $10 in United States gold coin, which is $\frac{9}{10}$ pure, weigh 258 grains, and 1869 sovereigns, $\frac{11}{12}$ pure, weigh 40 lbs.?

AVERAGE OF PAYMENTS AND OF ACCOUNTS.

Averaging Accounts is the process of finding a time at which several sums due at different times, and not bearing interest, can be paid without loss to debtor or creditor.

There are two cases, viz.: I. When the *terms of credit* begin at the same time; and II. When the *terms of credit* begin at different times.

EXAMPLE 47. A. M. Brown & Co., sold to James McGregor a bill of goods on the following terms: $300 cash, $400 due in 1 month, $250 due in 2 months, and $175 due in 4 months. At what time might the indebtedness be discharged by one cash payment?

$300 for 0 mo.	= $1 for	0 mo.
$400 for 1 mo.	= $1 for	400 mo.
$250 for 2 mo.	= $1 for	500 mo.
$175 for 4 mo.	= $1 for	700 mo.
$1125		1600 mo.

It is evident that $400 for 1 mo. is the same as $1 for 400 mo. Also that $250 for 2 mo. is equal to $1 for 500 mo., and $175 for 4 mo. = $1 for 700 mo.

Hence the credit of the whole debt is equal to the credit of $1 for 1600 mo.

Now since $1 for 1600 mo. = $1125 for $\frac{1600 \text{ mo.}}{1125}$. Or 1 mo. 12 + days; the time required is 1 mo. 13 da.

EXAMPLE 50. Brown & Webb sold to Irwin & Sons bills of goods as follows: June 5, '96, $420 due in 3 months; July 25, '96, $280 due in 3 months; and Aug. 30, '96, $350 due in 2 months. Find the average time of payment.

$420 due Sept. 5, $280 due Oct. 25, $350 due Oct. 30.

Reckoning days from Sept. 5,—earliest date on which money is due.

$420 × 0	= 0 mo.
$280 × 50	= 14000 mo.
$350 × 55	= 19250 mo.
$1050	33250 mo.

33250 mo. ÷ 1050 = 31 + da.

32 days after Sept. 5 is Oct. 7.

EXERCISE 32.

When could the following non-interest bearing debts be paid at one time without loss :

1. $500 due in 1 month, $500 due in 2 months, and $800 due in 4 months.

2. $1500 cash, $3000 payable in 30 days, and $2000 payable in 90 days.

3. $3000 ; $\frac{1}{4}$ in 3 months, $\frac{1}{4}$ in 2 months, and the rest in 4 months.

4. $840 due May 1 ; $650 due June 15 ; $900 due July 18 ; $1275 due Aug. 1.

5. March 1, a bill of $300 for 60 days ; April 15, $400 for 30 days ; June 10, a bill of $583.50 for 4 months ; July 1, $962.80 for 3 months.

6. Jan. 25, $850 on 4 months ; Feb. 15, $600 on 3 months ; March 20, $500 on 4 months ; April 10, $960 on 2 months.

7. A man owes $1000 to be paid at the end of 9 months without interest. How much would he require to pay at the end of 4 months to extend the time of payment 3 months ?

8. Henry Jones owes Peter Grant for goods as follows:

May 3, bill of $200 on 3 mo. credit.
May 24, " " $150 " 4 mo. "
June 20, " " $320 " 2 mo. "
July 15, " " $400 " 4 mo. "

On July 30, Peter Grant got Henry Jones' note for the amount, in one payment, when due. Mr. Grant, the same day, had the note discounted at 6% at the bank of Nova Scotia. What did he receive ?

EXAMPLE 51. In the books of Frank Calder & Sons is the account given below. What would be the date of an interest-bearing note given to settle the account?

DR. BASIL BELL. CR.

1897.				1897.			
July 1	To Mdse.....	$450	00	Aug. 8	By Cash.....	$200	00
July 10	" " 30 da..	300	00	Sept. 10	" Draft.....	350	00
Aug. 3	" " 2 mo.	550	00	Oct. 12	" " 10 da.	400	00

DR. TAKING OCT 25TH AS THE FOCAL DAY. CR.

Due.	Days.	Items.	Product.	Due.	Days.	Items.	Product.
July 1...	116	450	52200	Aug. 8..	78	200	15600
Aug. 9..	77	300	23100	Sept. 10.	45	350	15750
Oct. 3..	22	550	12100	Oct. 25..	0	400	0
		1300	87400			950	31350
		950	31350				
Balance		350	56050				

56050 ÷ 350 = 160 + days, average time of interest.

Oct. 25 – 161 days = May 17, date of note given by Mr. Bell.

Explanation:—Adding 3 days of grace to the date of maturity of the 10-day draft we get Oct. 25 as the latest date, and this we fix as our focal date. On Oct. 25, leaving the credit side of the acc. out, Basil Bell owes Frank Calder & Sons $1300, with interest on $1 for 87400 days. On the same day, leaving out the debtor side of the acc., Frank Calder & Sons owe Basil Bell $950 with interest on $1 for 31350 days. Subtracting the less from the greater we find that Basil Bell owes Frank Calder & Sons $350 with interest on $1 for 56050 days. Now the interest on $1 for 56050 days is equal to the interest on $350 for 161 days. Therefore the note for $350, with interest, must be dated 161 days anterior to Oct. 25; that is, on May 17, '97.

EXAMPLE 52. One of the accounts in George Metcalfe & Co.'s books is as below: On what date should a note given in settlement begin to bear interest?

DR. BLACK BROS. CR.

1898.				1898.			
Jan. 10	Mdse. 3 mo..	220	00	Jan. 12	Mdse. 2 mo..	500	00
Jan. 21	"	500	00	Jan. 26	" 1 mo..	435	00
Mar. 2	" 1 mo..	370	00	Feb. 20	Cash.	100	00
Mar. 12	Draft 30 da..	400	00	Ap. 25	Draft	380	00
Ap. 17	Mdse. 2 mo..	600	00	May 1	Mdse	520	00
				May 20	"	150	00
				June 2	"	200	00

DR. TAKING JUNE 17 AS FOCAL DAY. CR.

Due.	Days.	Items.	Product.	Due.	Days.	Items.	Product.
Ap. 10	68	220	14960	Mar. 12	97	500	48500
Jan. 21	147	500	73500	Feb. 26	111	435	48285
Ap. 2	76	370	28120	Feb. 20	117	100	11700
Ap. 17	64	400	25600	Ap. 25	53	380	20140
June 17	0	600	0	May 1	47	520	24440
				May 20	28	150	4200
				June 2	15	200	3000
		2090	142180			2285	160265
						2090	142180
						195	18085

18085 da. ÷ 195 = 92 + days.

On June 17 George Metcalfe & Co. would have to pay Black Bros. $195 with interest for 93 days, or give them a note, bearing interest, dated March 16, 1898.

EXAMPLE 52. When, in equity, should the balance of the following account be paid?

DR. JOHN C. CHAMBERS. CR.

1898.				1898.			
May 5	To Mdse....	50	00	May 15	Mdse	25	00
June 7	" " 2 mo.	140	00	June 10	Draft 10 da..	100	00
June 21	" " 30 da.	150	00	June 30	Cash	100	00

SELECTING AUG. 7 AS FOCAL DAY.

Due.	Days.	Items.	Product.	Due.	Days.	Items.	Product.
May 5	94	50	4700	May 15	84	25	2100
Aug. 7	0	140	0	June 23	45	100	4500
July 21	17	150	2550	June	38	100	3800
		340	7250			225	10400
		225					7250
		115					3150

3150 da. ÷ 115 = 27 + days.

Aug. 7 + 28 days = Sept. 4.

In this example the debit side of the acc. shows that J. C. Chambers owes $340 and the interest on $1 for 7250 days. The credit side shows that there is owing to him $225 and the interest on $1 for 10400 days.

Subtracting we find that he owes $115 *less the interest on $1 for 3150 days.* This latter is equal to the interest on $115 for 28 days. Hence he is not required to pay the $115 until 28 days after Aug. 7.

EXERCISE 33.

1. When should interest begin on the balance of the following account?

ANDERSON ROGERS IN ACC. WITH JAMES MCKENZIE.

DR.				1898.				CR.
1898.								
Feb. 20	To Mdse. 1mo	380	00	Mar. 25	By Cash.....	100	00	
Mar. 19	" Draft 30da	175	00	May 3	" Mdse 2mo	90	00	
Ap. 10	" Cash.....	120	00	May 15	" Mdse. 3mo	125	00	
May 4	" Mdse. 30da	650	00	June 18	" Cash.....	540	00	
				July 16	" Cash.....	140	00	

2. A man bought on Sept. 14, '97, $400 worth of goods, at 6 months' credit. On Nov. 25 he paid $115, and on Dec. 10 he paid $96. When, in equity, should he pay the balance?

3. On July 8, John Stewart gave John R. Fitzpatrick mdse. worth $365 on 30 da.; on Sept. 2, goods worth $300; on Sept. 12 a 30-day draft for $180; and on Oct. 4 mdse. worth $250 on 1 mo. Fitzpatrick gave Stewart on Sept. 24, cash $200; Oct. 1, cash $100; and Dec. 1 a 10-da. draft for $525. When, in equity, should Fitzpatrick pay the balance?

4. A man sold a farm for $2400. He received $200 cash, and was to receive $200 each month till all was paid. The purchaser, however, made no other payment until the end of 5 months when he paid $1000, and at the end of 7 months he paid $800. When could the balance be paid without loss to either party?

5. Alfred Dickie, on Jan. 1, 1896, gave Richmond Logan mdse. worth $500 on 1 mo.; on Jan. 20, mdse. $850 on 3 mo.; Feb. 15, mdse. $1500 on 2 mo.; and April 8, mdse. $2500 on 4 mo. On Feb. 3, Logan gave Dickie cash $500; on Feb. 28, cash $200; and on May 16, a 30-day draft for $1200. When could the balance be equitably paid in one sum?

WORK PROBLEMS.

EXAMPLE 54. A can do a piece of work in 8 days, working 10 hours a day, and B can do it in 6 days, working 12 hours a day; in how many days of 9 hours each can they together do the same work?

A does $\frac{1}{80}$ and B $\frac{1}{72}$ in 1 hour.

Working together A & B do $\frac{1}{80}+\frac{1}{72}$ or $\frac{19}{720}$ in 1 hour
" " A & B do $\frac{1}{720}$ in $\frac{1}{19}$ of an hour
" " A & B do $\frac{720}{720}$ in $\frac{720}{19}$ hours
$\frac{720}{19}$ hours $\div 9 = 4\frac{4}{19}$ days of 9 hours each.

EXAMPLE 55. A cistern can be filled by a pipe in $8\frac{1}{3}$ hours, and emptied by another in $13\frac{1}{3}$ hours; if both pipes be opened in what time will the empty cistern be filled?

First pipe fills $\frac{1}{8\frac{1}{3}}$ or $\frac{3}{25}$ in 1 h.

Second pipe empties $\frac{1}{13\frac{1}{3}}$ or $\frac{3}{40}$ in 1 h.

Working together they fill $\frac{3}{25}-\frac{3}{40}$ or $\frac{9}{200}$ in 1 hour
" " " " $\frac{1}{200}$ in $\frac{1}{9}$ of an hour
" " " " $\frac{200}{200}$ in $\frac{200}{9}$ or $22\frac{2}{9}$ hours

EXERCISE 34.

1. A can do a piece of work in 12 days, B in 15 days and C in 20 days; what fraction of the work can they do together in 3 days?

2. A cistern has three pipes, two of which can fill it in 5 and 6 hours respectively, while the third can empty it in 4 hours; if the three be opened when the cistern is empty how long will it take to fill it?

3. A can do a piece of work in 20 days and B in 25 days. A works at it for 15 days; in what time can B finish it?

4. A can do a piece of work in 20 days and B in 30 days. A works at it for 5 days and then stops work. B then works at it for 15 days and stops work. C afterwards finishes it in 5 days. In how many days could C do the work alone?

5. A does $\frac{7}{10}$ of a piece of work in 14 days. He is then joined by B and they finish the work in 2 days. How long would B take to finish the work by himself?

6. A can do a piece of work in 10 days, A and C in 7 days, and A and B in 6 days; in how many days can B and C, working together, do it?

7. A can do $\frac{1}{3}$ of a piece of work in 5 days, B $\frac{2}{5}$ in 4 days and C $\frac{5}{8}$ in 10 days; how long will it take them all working together?

8. A can do a piece of work in 10 days, working 8 hours a day. B can do the same work in 9 days working 12 hours a day. They work together and finish it in 6 days. How many hours a day do they work?

9. A certain sum of money pays the wages of two men for $10\frac{1}{2}$ days. It would pay the wages of one of them for $18\frac{3}{8}$ days. For how many days would it pay the wages of the other?

10. A can do $2\frac{1}{2}$ times and B $1\frac{1}{3}$ times as much work as C in a day. A and C work for 10 days on a job which they could finish in 12 days; A is taken off and B put on in his place; how many days does it take to do the whole work?

CLOCK PROBLEMS.

EXAMPLE 56. At what time between two and three o'clock are the hands of a watch together?

The circle of the face of the watch is divided into 60 minute divisions. At two o'clock the minute hand is at twelve, and is 10 minute-divisions behind the hour hand, which is at two. As the minute hand travels 12 times as fast as the hour hand it gains 11 minutes on the hour hand in every 12 minutes.

11 minute-divisions are gained in 12 minutes
1 " " is " in $\frac{12}{11}$ "
10 " " are " in $\frac{12 \times 10}{11} = 10\frac{10}{11}$ min

Ans. $10\frac{10}{11}$ min. past 2.

EXERCISE 35.

Find the time the hands of a clock are together between the hours of

1. 3 and 4. 2. 6 and 7. 3. 5 and 6.
4. 9 and 10. 5. 8 and 9. 6. 10 and 11.

EXAMPLE 57. At what times between 4 and 5 are the hands of a clock at right angles?

To be at right angles the hands must be 15 minute-divisions apart. As the hour hand is at 4 when the minute hand is at 12, the latter is 20 minute-divisions behind. When it gains 5 minute-divisions it will be at right angles.

11 minute-divisions are gained in 12 minutes
1 " division is " in $\frac{12}{11}$ "
5 " divisions are " in $\frac{12 \times 5}{11}$ " or $5\frac{5}{11}$ min.

Ans. $5\frac{5}{11}$ minutes after 4.

It is evident that after the minute hand has passed the hour hand it will be again at right angles. It will then have gained 35 minute-divisions on the hour hand. This worked out as above gives 38 $\frac{2}{11}$ min. past 4 as the second answer.

At what time are the hands of a clock at right angles between

7. 6 and 7. 8. 7 and 8. 9. 11 and 12.

NOTE.—When the hands of a clock are in a straight line they are 30 minute-divisions apart.

At what time are the hands of a clock in a straight line between

10. 1 and 2. 11. 5 and 6. 12. 8 and 9.

RATIO.

Ratio is the relation of magnitude in which one number stands to another.

The ratio of one number to another is briefly expressed by arranging them as dividend and divisor, or as numerator and denominator of a fraction.

The sign of ratio is (:), which is a modification of ÷, the sign of division. Thus the ratio of 4 to 9 is expressed

$$4 : 9,\ 4 \div 9,\ \text{or}\ \tfrac{4}{9}.$$

Its value is $\frac{4}{9}$. 9 : 4 is also a ratio. It is read "the ratio of 9 to 4." Its value is $\frac{9}{4}$ or $2\frac{1}{4}$.

The first Term of a ratio is called the Antecedent, and the second Term is called the Consequent.

A ratio can exist between concrete numbers only when they are of the same denomination; for example 8 lb. : 17 lb. equals 8 : 17.

A Simple Ratio is the ratio between two terms; as, 4 : 12.

Two or more simple ratios are compounded by multiplying the antecedents together for a new antecedent and the consequents for a new consequent.

$$\left.\begin{matrix} 3 : 6 \\ 4 : 5 \\ 5 : 8 \end{matrix}\right\} = 60 : 240.$$

The terms of a ratio may be multiplied or divided by the same number without changing its value; thus 60 : 240 = 120 : 480 (multiplying by 2), or = 30 : 120 (dividing by 2).

EXERCISE 36.

What is the value of the ratios :

1. $8 : 12$; $9 : 4$; $3 : 15$?

2. $9 : 16$; $8 : 3$; $36 : 4$?

3. $7 : 49$; $49 : 7$; $18 : 72$?

4. $6 : 7\frac{1}{2}$; $2\frac{1}{4} : 5$; $\frac{1}{3} : \frac{2}{5}$?

5. $\frac{3}{4} : \frac{3}{8}$; $3\frac{1}{3} : 10$; $8\frac{1}{4} : 17\frac{1}{2}$?

6. If the consequent is 16 and the value of the ratio is 4, what is the antecedent ?

7. If the antecedent is $14\frac{1}{2}$ and the ratio 3, what is the consequent ?

8. Compound the ratios in each of the first five questions of this exercise, and reduce to their simplest form.

EXAMINATION PAPER No. 13.

1. Simplify $4.0\dot{3} + 1.\dot{0}\dot{2} + 6.0\dot{1}\dot{8}$.

2. Find the cube root of $49.\dot{2}9\dot{6}$.

3. A note when drawn at two months, is discounted at 6%. The proceeds are \$500. What is the face of the note ?

4. What is the difference between 1 A. 160 sq. rd. 30 sq. yd. 2 sq. ft. 72 sq. in. and 2 A.

5. Bought 60 lb. of tea at 30c., 90 at 40c. and 50 at 50c. a lb. At how much a lb. must I sell the mixture to gain $33\frac{1}{3}$%?

EXAMINATION PAPER No. 14.

1. A man left $\frac{2}{5}$ of his money to his elder son, and $\frac{1}{3}$ of it to his brother and the remainder to his daughter. The elder son received \$2000 more than his sister. How much did each receive ?

2. What is the change in income when \$10,000 in 6% stock is sold out at $90\frac{1}{4}$, brokerage $\frac{1}{8}$%, and the proceeds lent on mortgage at $5\frac{1}{2}$% ?

3. When 4 per cent. bonds sell for $125\frac{1}{8}$, brokerage being $\frac{1}{8}\%$, what rate of interest does an investor get for his money?

4. Simplify $.\dot{4}\dot{6} \times .\dot{9} \div .3\dot{7}\dot{8}$.

5. Find the G. C. D. of 317634951 and 31383684951.

PROPORTION.

A **Proportion** is a statement of equality between ratios, which are expressed in different terms.

Thus, the ratios 5 : 15 and 3 : 9 being each equal to $\frac{1}{3}$ form a proportion.

A proportion may be indicated in the following ways:

Thus, 4 is to 8 as 12 is to 24 may be written:

$$4 : 8 : : 12 : 24,$$

or $4 \div 8 = 12 \div 24,$

or $\frac{4}{8} = \frac{12}{24}$.

The four points (: :) represent the extremities of the lines (=) used as the sign of equality.

Every proportion must consist of at least four terms; for it involves at least two equal ratios.

The first and third terms of a proportion are called the **Antecedents** (the antecedents of the ratios).

The second and fourth terms are called the **Consequents** (the consequents of the ratios).

When numbers form a proportion each ratio is called a couplet, and each term a proportional. Thus in the proportion, 5 : 10 : : 8 : 16, there are two couplets and four terms or proportionals.

In a proportion of four terms the first and last terms are called the **Extremes** and the second and third the **Means.**

The product of the extremes is equal to the product of the means.

Thus, in the proportion 4 : 8 : : 10 : 20, $4 \times 20 = 8 \times 10$.

Therefore if any three terms of a proportion be given the fourth may be found.

EXERCISE 37.

EXAMPLE 58. Find the missing term in the proportion :

$$8 : 16 : : (\quad) : 40.$$

Since $8 \times 40 = 320$, the product of the extremes, $16 \times$ by the missing term $= 320$, the product of the means.

Therefore $320 \div 16 =$ the missing term $= 20$.

Find the missing term in each of the following :

1. $4 : 8 : : (\quad) : 32$; $10 : 15 : : (\quad) : 60$.
2. $3 : (\quad) : : 18 : 36$; $2\frac{1}{4} : (\quad) : : 5 : 15$.
3. $(\quad) : 18 : : 20 : 10$; $(\quad) : 7\frac{1}{2} : : \frac{6}{8} : \frac{3}{8}$.
4. $5 : 12 : : 16 : (\quad)$; $14 : 20 : : 1\frac{1}{3} : (\quad)$.
5. $\$6 : \$18 : :$ 9 yd. $: (\quad)$; £4 : 10s. 6d. : : 1 cwt : ( ).
6. 3 lb. 12 oz. : 11 lb. 4 oz. : : \$3.50 : ( ).
7. 1 a. 30 rods : 38 rods : : £5 10s. : ( ).
8. $\frac{2}{3} : .05 : : .\dot{0}\dot{5} : (\quad)$.

EXAMPLE 59. If 5 lb. of butter cost \$1.20, what will 15 lbs. cost?

It is evident that the ratio

5 lb. : 15 lb. = the ratio, \$1.20 the price of 5 lb. : the price of 15 lb. Therefore we have the proportion :

$$5 : 15 : : \$1.20 : (\quad).$$

$$\text{Or, } \frac{\$1.20 \times 15}{5} = \$3.60 \text{ Ans.}$$

EXAMPLE 60. If 5 lb. of butter cost \$1.20, how much can be bought for \$3.60.

$$\$1.20 : \$3.60 : : 5 \text{ lb.} : (\quad).$$
$$\text{Or, } \quad 120 : \quad 360 : : 5 \qquad (\quad).$$

$$\frac{360 \times 5}{120} = \frac{1800}{120} = 15 \qquad \text{Ans. 15 lb.}$$

EXERCISE 38.

1. If a man can travel 5 miles in 20 hours, how far could he travel in 10 days walking 10 hours a day?

2. If 19 meters of tweed cost $57.57, how many yards can be bought for $460.56? (A meter = 39.37 in.)

3. 8 acres 160 rods of pasture were sold for $460. How many acres could be bought for $3680?

4. I bought drugs at $16 a lb. avoirdupois. At how much a dram apothecaries' weight must I sell to gain 80%?

5. If 120 men consume 40 bbls. of flour in a certain time, how many men will consume 1600 bbls. in the same time?

6. Paid $10.50 for 11 lb. 4 oz. of tea; what should I pay for 3 lb. 12 oz.?

7. An insolvent fails for $7000, and his assets amount to $3000. What does a creditor receive to whom he owes $450?

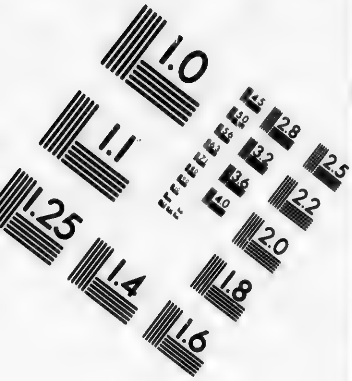
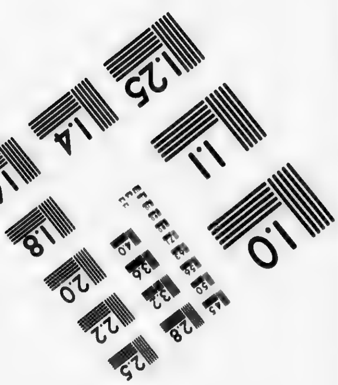
8. If 2 lb. of sugar cost 25 cents, and 8 lb. of sugar are worth 5 lb. of coffee, what will 50 lb. of coffee cost?

9. A grocer has a false balance by which 1 lb. will weigh but 12 oz. He sells a barrel of sugar for $28. How much more than the real value does the customer pay?

10. A dealer in selling cheese gives only $14\frac{11}{16}$ oz. for a pound. A customer, who buys $30 worth pays how much more than he ought?

11. A has land that he values at $50 an acre, and B land at $56 an acre. If in trade A gets $56 an acre what should B get to do as well?

12. If 3 men can do $\frac{5}{12}$ of a piece of work in 5 days of 12 hours each, how many men will it take to do $\frac{4}{9}$ of the work in 6 days of 8 hours each?


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0

1.0
1.1
1.25
1.4
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0

SERIES.

A **Series** is a succession of numbers regularly
ing or diminishing according to a fixed law.

The numbers which compose the series are
Terms.

The following kinds of Series are called P
sions:

ARITHMETICAL PROGRESSION.

An **Arithmetical Progression** is a series wh
increase or diminish by a constant difference, c
Common Difference. For example: 2, 4, 6, 8,
Ascending Series in which the Common Differe
21, 18, 15, 12, 9, is a Descending Series, in w
Common Difference is 3.

The first and last terms of a series are c
Extremes.

The terms between the first and last are c
Means.

In an arithmetical progression there are five
to be noted—the first term, the last term, the comm
ence, the number of terms, and the sum of the te

If any three of these be given the other tw
found.

To find any Term of an Arithmetical Progr

EXAMPLE 61. Find the 12th term of the pr
3, 7, 11, 15, etc.

Com. diff. $= 7 - 3 = 4$. The common diffe
12th term $= 3 + (11 \times 4)$ The *second* term is
$= 3 + 44 = 47$. the first term plus
common difference; the third term is equal to
term plus *twice* the common difference; the *twe*
is equal to the first term plus eleven times the
difference.

PROGRESSION.

mbers regularly increas-
fixed law.
the series are called its

ies are called **Progres-**

OGRESSION.

on is a series whose terms
nt difference, called the
ple : 2, 4, 6, 8, 10, is an
Common Difference is 2.
ng Series, in which the

f a series are called the

t and last are called the

on there are five elements
t term, the common differ-
he sum of the terms.
en the other two may be

rithmetical Progression :

h term of the progression

The common difference is 4.
he *second* term is equal to
e first term plus *once* the
term is equal to the first
ference ; the *twelfth* term
eleven times the common

EXAMPLE 62. Find the tenth term of the progression 100, 96, 92, etc.

$100 - 96 = 4$ The common difference is

10th term $100 - (9 \times 4)$ 4. The *second* term is equal

$= 100 - 36 = 64$ to the first term minus once the common difference ; the *third* term is equal to the first term *minus* twice the common difference ; the *tenth* term is equal to the first term minus nine times the common difference.

EXERCISE 39.

1. Find the 19th term of the series 4, 7, 10, etc. .

2. What is the 13th term of the progression 75, 70, 65, etc.?

3. A man sold a dozen of eggs, getting 1 cent for the first, 3 cents for the second, 5 for the third. Continuing at that rate what did he get for the last egg ?

4. What is 25th term of the series $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, etc.?

5. The first term of a series is 6, the common difference is 4, and the number of terms 11. What is the last term ?

6. Find the 9th term in the series $1\frac{7}{8}$, $1\frac{3}{4}$, $1\frac{5}{8}$, $1\frac{1}{2}$, etc. .

7. The first term of an ascending series is $6\frac{1}{2}$, the common difference $\frac{3}{4}$, and the number of terms 15 ; what is the last term ?

8. The first term of a descending series is 1, the common difference .05, and the number of terms 15. Find the last term.

To find the Sum of the Terms of an Arithmetical Progression :

EXAMPLE 63. Find the sum of the terms of the progression 2, 6, 10, 14, 18, 22, 26, 30.

Sum of the terms $= 2 + 6 + 10 + 14 + 18 + 22 + 26 + 30.$

Also sum of the terms $= 30 + 26 + 22 + 18 + 14 + 10 + 6 + 2.$

Therefore twice the sum $= 32 + 32 + 32 + 32 + 32 + 32 + 32 + 32.$

That is, twice the sum $=$ 8 times 32.

And the sum $= \frac{8 \text{ times } 32}{2} = \frac{8 \times 32}{2} = 128.$

Since 32 is the sum of the first and the last term, and 8 is the number of terms, we find that the sum of the terms of an arithmetical progression is obtained by multiplying the sum of the first and last terms by the number of terms, and dividing the result by 2.

EXAMPLE 64. Find the sum of the progression 3, 8, 13, 18, etc., to 20 terms.

$$\text{The last term (20th)} = 3 + (19 \times 5) = 98$$

$$\text{The sum} = \frac{(3 + 98) \times 20}{2} = 1010.$$

EXERCISE 40.

1. Sum the series 3, 6, 9, 12, etc., to 10 terms.
2. " " " 60, 54, 48, etc., to 12 terms.
3. " " " 2, $2\frac{3}{5}$, $3\frac{1}{5}$, $3\frac{4}{5}$ to 21 terms.
4. " " " 100, $96\frac{2}{3}$, $93\frac{1}{3}$, 90 to 31 terms.
5. " " " $\frac{2}{3}$, $\frac{5}{6}$, 1, etc., to 50 terms.
6. Find the sum of the integers, beginning with one and ending with 100.
7. A body falls, approximately, 16 feet in the first second, and in each succeeding second 32 feet more than in the preceding one. How far will it fall in 20 seconds, and what distance will it fall in the last second?
8. A man sold 90 sheep, receiving 10 cents for the first, 20 cents for the second, getting ten cents more for each sheep, than for the preceding one sold. What did he get for the lot?

GEOMETRICAL PROGRESSION.

A Geometrical Progression is a series of numbers which increase or decrease by a constant multiplier, called the Common Ratio.

Thus: 1, 3, 9, 27, etc., is an ascending geometrical progression in which the common ratio is 3;

And 64, 32, 16, 8, etc., is a descending geometrical progression in which the common ratio is $\frac{1}{2}$.

In an ascending geometrical progression the common ratio is more than unity; in a descending geometrical progression the common ratio is less than unity.

The numbers which compose the progression are called its **Terms.**

In a geometrical progression there are five elements to be considered, any *three* of which being given, the other two may be found. They are the *first term*, the *last term*, the *common ratio*, the *number of terms*, and the *sum of the terms.*

The first and last terms are called the **extremes** and the intermediate terms the **mean.**

To find any Term of a Geometrical Progression:

EXAMPLE 65. Find the sixth term of the geometrical progression 2, 6, 18, etc.

Ratios $= 6 \div 2 = 3$ Dividing the second term, 6,
6th term $= 2 \times 3^5$ by the preceding term, 2, the
$= 2 \times 243$ common ratio is found to be 3.
$= 486$ Ans. Since the *second* term is equal to the product of the first term and the first power of the ratio (2×3), and the *third* term is equal to the product of the first term and the second power of the ratio (2×3^2), etc.;

Therefore the *sixth* term will be equal to the product of the first term and the fifth power of the ratio, (2×3^5).

To find any term of a geometrical progression, multiply the first term by that power of the ratio indicated by the number of terms less 1.

EXERCISE 41.

1. Find the 8th and 9th terms of the geometrical progression 2, 4, 8, 16, etc. .

2. Find the 8th and 10th terms of the geometrical progression 1, 4, 16, 64, etc. .

3. Find the 12th and 16th terms of the series $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, etc. .

4. Find the 6th and 11th terms of the series 100, 50, 25, $12\frac{1}{2}$, etc. .

5. The first term of a geometrical progression is $\frac{3}{8}$ and the common ratio $1\frac{1}{3}$. Find 6th, 8th and 10th terms.

6. The first term of a series is 1, and the common ratio is $\frac{1}{4}$; what is the 8th term?

7. The *sixth* term of a geometrical progression is 3888, and the ratio 6; find the first term.

8. The last term of a geometrical series is $60\frac{3}{4}$, the ratio $\frac{3}{4}$, and the number of terms 7; what is the first term?

To find the Sum of a Geometrical Progression:

EXAMPLE 66. Find the sum of the geometrical progression 2, 8, 32, 128, 512.

4 times the sum = $8 + 32 + 128 + 512 + 2048$ (1)

1 time the sum = $2 + 8 + 32 + 128 + 512$ (2)

By subtracting (2) from (1) we have

3 times the sum = $2048 - 2$

1 time the sum = $\frac{2048 - 2}{3} = 682$

Now $2048 = 512 \times 4$ = the greater extreme multiplied by the ratio

2 = less extreme

3 = the ratio minus 1.

Therefore to find the sum of a geometrical series, multiply the greater extreme by the ratio, subtract the less extreme from the product, and divide the result by the ratio minus one.

When the series is a descending one, write it in the reverse order, thus making the ratio more than one.

NOTE.—By writing a descending series in the reverse order it becomes an ascending series. The ratio is then the reciprocal of the ratio of the descending series.

Thus: 64, 32, 16, 8, 4 whose ratio is $\frac{1}{2}$,
becomes 4, 8, 16, 32, 64 whose ratio is 2.

EXERCISE 42.

1. Find the sum of the series 3, 12, 48, etc., to 6 terms.

6th term $= 3 \times 4^5 = 3072$ (Art.

$$\frac{3072 \times 4 - 3}{4 - 1} = 4095$$

2. Find the sum of the geometrical series 162, 54, 18, etc., to 5 terms.

5th term $= 162 \times (\frac{1}{3})^4 = 2.$

By writing in the reverse order the series becomes

2, 6 . . . . 162 whose ratio is 3 (reciprocal of $\frac{1}{3}$)

$$\text{Sum} = \frac{162 \times 3 - 2}{3 - 1} = 242$$

3. Find the sum of the progression 1, 4, 16, etc., to 10 terms.

4. Find the sum of the progression $\frac{3}{4}$, $\frac{3}{8}$, $\frac{3}{16}$ to 8 terms.

5. " " " " " 120, 60, 30, etc., to 7 terms.

6. Find the sum of the progression 1, 2, 4, etc., to 11 terms.

7. Find the sum of the progression 20, 10, 5, etc., to 10 terms.

8. Find the sum of the progression $\frac{3}{4}$, $\frac{9}{16}$, $\frac{27}{64}$, etc., to 6 terms.

9. If a boy saves one cent on Monday, 2 cents on Tuesday, 4 cents on Wednesday, and so on for 7 days, how much will he save in the week?

10. A man in business doubled his capital every 5 years, and began with a $1000. How much has he at the end of 25 years?

11. A sum of money is to be paid in 10 instalments, the first of which is $512, the second $256, the third $128 and so on. What is the sum?

An **Infinite Series** is a descending series of an infinite number of terms. Thus: 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, etc., is an infinite series, the last term of which is infinitely small and is regarded as 0.

EXAMPLE 67. Find the sum of the infinite series 9, 6, 4, etc.

This is a descending series the last term of which is 0 and the ratio $\frac{2}{3}$. By reversing the series we have 0, . . . 4, 6, 9, with the ratio $\frac{3}{2}$,

Then $$\frac{9 \times \frac{3}{2} - 0}{\frac{3}{2} - 1} = 27.$$

EXERCISE 43.

Sum to infinity the following series:

1. 12, 6, 3, etc.
2. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc.
3. 12, 4, $1\frac{1}{3}$, etc.
4. 500, 100, 20, etc.
5. 768, 192, 48, etc.
6. 6, 1, $\frac{1}{6}$, etc.
7. 100, 10, $\frac{1}{10}$, etc.
8. 1, $\frac{1}{9}$, $\frac{1}{81}$, etc.
9. .5, .05, .005, etc.
10. $.\dot{3}\dot{6}$.

COMPOUND INTEREST.

Problems in compound interest may be solved by means of the principles of geometrical progression.

Thus: Suppose \$100 be put out at compound interest at 5%.

The amount at the end of one year = $\$100 \times 1.05$.

The amount at the end of two years = $\$100 \times 1.05 \times 1.05$ or $\$100 \times 1.05^2$.

The amount at the end of three years = $\$100 \times 1.05 \times 1.05 \times 1.05$ or $\$100 \times 1.05^3$; and so on.

To find the amount of a sum of money at compound interest for any number of years, multiply the principal by that power of the sum of 1 and the rate, whose exponent is equal to the number of years.

EXAMPLE 68. Find the amount of \$250 for 3 years at 5% compound interest.

$\$250 \times (1.05)^3 = \$289.406250$ or, \$289.41.

This question makes a geometrical progression of four terms, the first term being 250 and the common ratio 1.05. It is required to find the fourth term.

EXERCISE 44.

Find the amount and compound interest of:

1. \$440 for 3 years at 6%.
2. \$1000 for 4 years at 3%.
3. \$690 for 2 years at 5%.
4. \$300 for 6 years @ 4%.
5. \$2100 for 5 years @ 2%.
6. \$1800 for 2 years @ $1\frac{1}{2}$%.

ANNUITIES.

An **Annuity** is a sum of money payable yearly, to continue a number of years, for life, or forever.

The term is also applied to money payable at any regular times.

The **Amount** or **Final Value** of an annuity is the sum of all the payments, together with the interest on each payment from the time it becomes due until the annuity ceases.

The **Present Worth** of an annuity is that sum of money which, at the specified rate of interest, will amount to the final value.

ANNUITIES OF SIMPLE INTEREST.

EXAMPLE 69. Find the amount of an annuity of $300 for 5 years at 6%, simple interest.

The first payment draws interest for four years; the second for 3 years; etc.

Now the amount of the first payment at the end of the four years during which it draws interest, is $300 × 1.24 or $372.

The amount of the second payment at the end of the three years during which it draws interest, is $300 × 1.18.

The amount of the last payment is $300.

This makes an arithmetical progression of 5 terms, the first term of which is $372, the last term $300, and the number of terms 5.

Therefore the sum of the terms (amount or final value of the annuity) is

$$\frac{(\$372 + \$300) \times 5}{2} = 1680.$$

EXAMPLE 70. Find the present worth of an annuity of $400 for 6 years at 5% simple interest.

The amount of the annuity is $2700.

Amount of $1 in 6 years at 5% = $1.3.

Principal to produce $1.3 = $1

" " " $1 $= \frac{\$1}{1.3}$

" " " $2700 $= \frac{\$1 \times 2700}{1.3} = \$2076.92.$

Example 71. What annuity to continue for 4 years, at 6% simple interest, can be purchased for $2180?

An annuity of $1 to continue for 4 years, at 6% simple interest, will amount to

$$\frac{(\$1.18 + \$1) \times 4}{2} \text{ or } \$4.36.$$

The amount of $2180 for 4 years, at 6% is $2703.20.

$4.36 is the amount or final value of an annuity of $\frac{\$1}{4.36}$

$1 " " " " " " $\frac{\$1}{4.36}$

2703.20 " " " " " " $\frac{\$1 \times 2703.20}{4.36}$ or $620

EXERCISE 44a.

1. What are the present worth and final value of an annuity of $500 for 5 years, at 6% simple interest?

2. What are the amount and present worth of an annuity of $600 for 5 years, at 4% simple interest?

3. Find the amount and present worth of an annuity of $500 for 3 years, at $4\frac{1}{2}$% simple interest.

4. Find the amount and present worth of an annuity of $800 for 8 years, at 5%.

5. What annuity to continue for 8 years, at 3% simple interest can be purchased for $2088?

6. Find the amount and present worth of an annuity of $720 for 7 years, at $3\frac{3}{4}$% simple interest.

7. What annuity to continue for 6 years, at 6% simple interest can be bought for $2070?

8. What is the annuity whose amount for 6 years, at 6% simple interest is $3450?

9. What annuity to continue for 9 years, at 4% simple interest, can be purchased for $1044?

10. $11220 is invested in an annuity to continue 11 years, at $5\frac{1}{2}$% simple interest. What is the yearly payment?

ANNUITIES AT COMPOUND INTEREST.

EXAMPLE 72. Find the amount of an annuity for \$400 for 5 years at 4% compound interest.

The fifth or last payment is \$400 and draws no interest.

The fourth or second last payment is \$400 and draws interest for 1 year and therefore *amounts* to \$400 × 1.04.

The third payment amounts to \$400 × 1.04 × 1.04 since it draws interest for two years, and so on.

The last payment may be regarded as the first term of an ascending geometrical progression of 5 terms whose common ratio is 1.04 and the sum of whose terms is equal to the amount of the annuity.

The fifth term (greater extreme) = \$400 × $(1.04)^4$.

$$\text{Sum of the terms} = \frac{(\$400 \times (1.04)^4) \times 1.04 - \$400}{1.04 - 1.}$$

$$= \frac{\$400 \times (1.04)^5 - \$400}{.04} = \$2166.53. \text{ Ans.}$$

EXAMPLE 73. Find the present worth of an annuity of \$400 for 5 years, at 4% compound interest.

The amount or final value of the annuity (see answer to preceding example) = \$2166.53.

Amount of \$1 for 5 years at 4% = $(1.04)^5$ = 1.2166529024.

Present worth of \$1.2166529024 = \$1

" " " \$1 $= \frac{\$1}{1.2166529024}$

" " " \$2166.53 $= \frac{\$1 \times 2166.53}{1.2166529024}$

= \$1780.73.

EXAMPLE 74. What annuity, to continue for 4 years, at 5% compound interest, can be purchased for $600?

Amount or final value of an annuity of $1 to continue for 4 years, at 5% compound interest is

$$\frac{(1.05)^4 - 1}{.05} \text{ or } \$4.310125.$$

Amount of $600 for 4 years, at 5% compound interest, is $\$600 \times (1.05)^4$ or $729.30375.

$4.310125 is the amount of an annuity of $1

$1 " " " " $\frac{\$1}{4.310125}$

$729.30375 " " " " $\frac{\$1 \times 729.30375}{4.310125}$

$= \$169.21.$

EXERCISE 45.

1. What are the amount and present worth of an annuity of $400 for 3 years, at 5% compound interest?

2. What is the final value of an annuity of $300 for 4 years, at 6% compound interest; and what sum will purchase such an annuity?

3. What annuity to continue for 2 years, at 6% compound interest, can be purchased for $5150?

4. What annuity to continue for 3 years, at 4% compound interest, can be bought for $3902?

5. The final value of an annuity for 5 years, at 6% compound interest is $2818.55; what is the annuity?

SQUARE ROOT.

(PART III., ARTICLES 57, 58).

EXERCISE 46.

1. An army of 220900 men is drawn up in the form of a solid square. How many men are there on each side?

2. The sides about the right angle of a right angled triangle are 72 and 96 feet respectively. What is the length of the hypotenuse?

3. A piece of land in the form of a square contains 10 acres. What is its perimeter?

4. The hypotenuse of a right angled triangle is 125 yards and one of the sides is 100 yards. What is the length of the other?

5. The end of a tree broken 39 feet from the top struck the ground 15 feet from the root. How many feet of the tree remained standing?

6. A ladder 40 feet long being placed at a certain point in the street will reach a window 33 feet from the ground on one side and a window 21 feet from the ground on the other. What is the width of the street?

7. A room is 40 feet long, 30 feet wide, and 15 feet high. What is the longest distance that can be measured in a straight line in the room?

8. The hypotenuse of a right angled triangle is 41^{m}, and one of the sides is $.9^{m}$. How many hektometers in the other side?

9. The sides of a right angled triangle are 18^{m} and 33^{dm}, respectively. Find the hypotenuse in dekameters.

10. Two vessels sail from the same place at the same time. One sails north for two days at the rate of 5 miles an hour and the other sails east for three days at the rate of 6 miles an hour. How far are they then apart?

MEASUREMENT OF AREAS.

(PART III., ARTICLES 60, 62, 63).

EXERCISE 47.

1. How many acres in a square field whose side is 24 rods?

2. What will it cost to paint the walls of a room 10 feet 9 inches long, 9 feet wide and 10 feet high at 25 cents a square yard?

3. The floor of a room 16 feet long and 12 feet wide is to be painted in alternate squares of black and white, the side of every square being 8 inches. If the white squares cost a cent each and the black $1\frac{1}{2}$ cents each, what will be the total cost?

4. A field contains 4 acres and is 70 yards long. What is its width?

5. What is the cost of carpeting a room 18 feet 3 inches long and 16 feet 4 inches wide with carpet 27 inches wide at $2.70 a yard?

6. How much must be paid for laying asphalt on $\frac{5}{8}$ of a mile of sidewalk at 75 cents a square yard, the sidewalk being 10 feet wide?

7. A contractor received $8000 for building a piece of road 800 yards long and 60 feet wide. How much did he receive a square yard?

8. A square yard of a floor costs $2.80 and the whole floor costs $114. What is its length if the breadth is $5\frac{1}{4}$ yards?

9. How many suits can be made out of 60 yards of tweed, 54 inches wide, each suit requiring 7 yards, 27 inches wide?

10. What will it cost to paper the walls of a room 19 feet 8 inches by 17 feet 9 inches and 12 feet high, with paper 18 inches wide, 7 yards in a roll, at 63 cents a roll?

11. What will it cost to paint a house 54 feet deep, and 24 feet 6 inches, fronting the street, at 20 cents a square yard, the walls being rectangular and 24 feet high?

12. What must be paid for the plastering of a wall 50.5 yards long and 5.25 yards high, at $2.10 a square yard?

13. There are 12 rooms in a school building, each room being 32 feet long, 24 feet wide and 12 feet high. What will it cost to paint the ceilings and walls of the whole building at 18 cents a square yard?

14. One piece of ground is 200 *yards square*, and another is 200 square yards. What is the difference in price at 45 cents a square foot?

15. What is the area of a triangle whose base is 40 feet and altitude 25 feet?

16. One side of a triangular field is 12 chains, 76 links, and the perpendicular on it from the opposite angle is 9 chains, 43 links. How many acres in it?

17. Find the area of a parallelogram, one of its sides being 40 inches and the shortest distance to the opposite side being 30 inches.

18. The area of a parallelogram is 1200 square yards, and the perpendicular distance between two opposite sides is 20 yards. What is the length of each of those sides?

19. It costs £12 to carpet a room 4 yards wide at $\frac{1}{5}$ of a pound a square yard; what is the length of the room?

20. What is the area of a trapezium whose parallel sides are 35 and 45 feet, respectively, and altitude 25 feet?

To find the area of a triangle when the three sides are given.

EXAMPLE 75. Find the area of a triangle whose sides are 42, 45 and 39 yards.

$$\frac{42+45+39}{2}=63$$

$$63(63-42)\,(63-45)\,(63-39)=571536$$

$$\sqrt{571536}=756$$

Ans. 756 sq. yds.

From half the sum of the three sides subtract each side separately; multiply half the sum and the three remainders together; the square root of the product is the area.

EXERCISE 48.

1. What is the area of a triangle whose sides are 60, 80 and 90 feet?

2. How many square yards in a triangular field whose sides are 126, 247 and 296 yards?

3. What is the area of an isosceles triangle whose base is 20 feet, and each of its equal sides 15 feet?

4. The sides of a field in the form of an equilateral triangle are each 70 rods. How many acres in it.

5. The sides of a triangular field are 1200, 1800 and 2400 links respectively. Find the area in acres and rods.

MEASUREMENT OF THE CIRCLE.

(PART III., ARTICLES 74, 75).

EXERCISE 49.

1. The diameter of a wheel is 26 inches. What is its circumference?

2. What is the circumference of a circle whose radius is 5 feet?

3. What is the length of a tire that will fit a wheel whose radius is 3 feet $10\frac{1}{2}$ inches?

4. The circumference of a circle is 15708 feet. What is its diameter?

5. What is the area of a circle whose diameter is 60 feet?

6. The radius of a circle is 17 feet 6 inches. Find its area.

7. The distance from the centre of a circular pond to the bank is 225 links. How many acres in it?

8. The circumference of a circle is 200 feet. Find its area.

9. The circumference of a circular field is 50 chains. What is its area?

10. The diameter of a circular garden is 40 rods. What is the area of a circular space 10 feet wide and just within its border?

11. The diameters of two circles having the same centre are 20 and 32 feet. Find the area of the circular space (the annulus) inclosed between the two circumferences.

12. A circular path 10 feet wide is laid at a distance of 40 feet from a certain statue in a public garden. What is its cost at 50 cents a square yard?

MEASUREMENT OF LUMBER.

(ART. 76, PART III.)

EXERCISE 50.

1. How many superficial feet in a board 36 ft. long, $1\frac{1}{2}$ ft. wide and $1\frac{1}{2}$ in. thick?

2. How many feet are there in nine joists, which are 15 ft. long, 5 in. wide, and 3 in. thick?

3. Find the number of (board) feet of lumber required to floor a dock 100 feet long and 40 feet wide, the planks being $2\frac{1}{2}$ inches thick.

If the board is tapering, take half the sum of its ends for the width.

4. What is the number of feet in a tapering piece of plank, 20 feet long, 24 inches wide at one end, and 16 inches wide at the other, the board being 2 in. thick?

5. How many feet in 3 beams 24 ft. long, 10 in. thick, whose width tapers from 18 to 16 inches?

6. How many board feet in a cubical block of wood whose edge is 2 feet 9 inches?

MEASUREMENT OF SOLIDITY OR VOLUME.

(ARTICLES 78-80, PART III.)

EXERCISE 51.

1. Find the volume of a cube whose edge is $3\frac{1}{4}$ inches, and also find the area of its entire surface.

2. What is the weight of a cubical piece of granite whose edge is 2 feet?

3. How many pounds avoirdupois in a piece of marble 2^m long, $.75^m$ wide and 8^{cm} thick?

4. How many kilograms in a block of ice 1 ft. long, 10 in. broad and 9 in. thick?

5. What will it cost to make an excavation 21 feet long, 18 feet broad and 10 feet deep at $1.30 a cubic yard?

6. The walls of the foundation of a house are 50 feet long, and 30 feet wide, measured on the outside. If they are 7 feet high and 9 inches thick, what will they cost at 30 cents a cubic foot?

7. What will be the cost of a pile of wood $35\frac{1}{2}$ feet long, $6\frac{1}{2}$ feet high, and 4 feet wide at $3.84 a cord?

8. How many granite blocks, each 8 in. long, $2\frac{3}{4}$ in. wide, and 2 in. thick, will be required to build a wall 18 ft. long, 3 ft. high, and 11 in. thick?

9. How many bushels of grain can be put into a bin 8 feet long, 3 feet wide and 4 feet deep?

10. How many tons of coal can be put into a bin 16 feet long, 5 feet 3 inches wide, and 6 feet 4 in. deep, a ton taking up 38 cubic feet of space?

11. A bin, which is 10 ft. long, and 3 ft. 6 in. wide, holds 100 bu. How deep is it?

12. A vessel 5 feet long, 4 feet wide, and 3 feet deep is filled with wheat. What is the weight of the wheat and what does it cost @ $1.00 a bushel?

(See Part III., Articles 81-90).

EXERCISE 52.

1. What is the volume of a cylinder whose altitude is 7 feet, and the diameter of base 6 feet?

2. The volume of a cylinder is 2412.7488 cu. ft., and its altitude is 12 feet. Find the radius of the base.

3. A cylindrical vessel is 28 feet high, and measures 5 feet across the base. How many cubic feet are in it?

4. The diameter of the base of a cone is 1 ft. 6 in., and the altitude 15 feet. What is its solidity?

5. The base of a marble pyramid is in the form of an equilateral triangle, each of whose sides is 3 feet. The altitude of the pyramid is 9 feet. What is its value at $2.50 a cubic foot?

6. Find the length of each side of the base of a square pyramid whose altitude is 21 feet and volume 847 cu. ft.

7. What is the volume of a square pyramid, the area of whose base is 36^{m}, and whose height is three times the diagonal of the base?

8. The diameter of a sphere is 25 inches. How many cubic inches are in it?

9. What is the weight of the hydrogen that will fill a spherical balloon, whose diameter is 3^{m}, the weight of a liter of hydrogen being $.09^{g}$?

10. What is the weight of a cast-iron ball whose diameter is 12 inches?

11. The diameter of a hollow sphere is $6\frac{1}{6}$ decimeters. How many liters will it hold?

12. The altitude of a cone is 20 inches, and the diameter of its base is 10 inches. The upper part is sawed off in a plane parallel to the base and 10 inches from it. How many cubic inches in each part? (The lower part is called the Frustum of the cone).

SPECIFIC GRAVITY.

EXERCISE 53.

(For Explanation See Part III., Examples 58, 59).

1. Find the specific gravity of a stone which weighs 21 lb. in air and 12.25 in water.

2. What is the specific gravity of a piece of marble which weighs 46.41 lb. in air and 17 lb. less in water?

3. If a substance weighs 25% less in water than in air, what is its specific gravity?

4. What is the specific gravity of a substance which weighed 25% more in air than it did in water?

5. What is the weight of a block of ordinary granite 1.6^{m} long, 7.5^{dm} wide and 4.5^{dm} thick?

6. If a bar of lead is 7^{m} long, 1^{dm} wide and 4^{cm} thick, what is its weight?

7. A bar of iron weighing 7.5 lb. in air and 6.5 in water is fastened to a piece of wood weighing 5 lb. in air. Together they weigh 3.6875 lb. in water. What is the specific gravity of the wood?

8. An empty glass bottle weighing 6.66 oz. is filled with olive oil weighing 7.32 oz. What is the specific gravity of the bottle of oil?

9. A body weighs 7.55 grams in air, 5.17 grams in water, and 6.35 in another liquid. Find from these data the specific gravity of the body in question, and also of the other liquid.

10. A block of wood floats with $\frac{7}{12}$ of its bulk out of the water. Find its specific gravity.

11. A pebble weighs 20 grams in air; immersed in water it weighs 15 grams; immersed in another liquid it weighs 17 grams. What is the specific gravity of the latter liquid?

12. A piece of wood weighs 7 lb. in air, and a piece of iron 7.8 lb. in air and 6.7 lb. in water. The wood and iron together weigh 5.3 lb. in water. Show that the specific gravity of the wood is $\frac{5}{6}$.

13. A solid, soluble in water, but not in alcohol, weighs 346 grains in air and 210 grains in alcohol. Find the specific gravity of the solid, that of alcohol being .85.

14. Find the specific gravity of a piece of wood from the following data:

Weight of wood in air = 25.35 lb.
" " a metal sinker = 11 lb.
" " wood and sinker in water = 5.1 lb.
Specific gravity of metal sinker = 8.95.

EXAMINATION PAPERS.

15.

1. What will eight hundred seventy-five thousandths of a ton cost, at $2.75 per ton?

2. Simplify $\dfrac{.05}{.09 \div .08}$

3. My purse and money are worth 12s. 8d., the money being worth 7 times the purse. How much is the purse worth.

4. If $\frac{3}{5}$ of a yard of satin lining is worth $\frac{3}{4}$ of a yard of silk lining, and silk lining is worth $\$\frac{2}{3}$ per yard, how many yards of satin lining will $20 buy?

5. Find the cube root of $\frac{5}{9}$ to 3 decimal places.

16.

1. Simplify $\dfrac{1}{3\frac{1}{2} + \dfrac{1}{4\frac{1}{2}}} \times .09$

2. I owned $\frac{5}{8}$ of a ship and sold $\frac{3}{4}$ of my share for $45000. What fraction of the whole ship had I left and what is its value at the same rate?

3. One insurance company offered to insure my house at $3\frac{1}{8}\%$, and another offered to insure it at $2\frac{7}{8}\%$. By accepting the latter offer I saved $10. How much insurance did I effect?

4. Multiply £1 17s. $6\frac{3}{4}$d. by $17\frac{3}{7}$.

5. I sent $4120 to my agent in Ontario to buy flour, after deducting his commission at 3%. How many barrels did he buy, flour being $5 a barrel.

17.

1. A man sold $\frac{1}{3}$ of his land and then $\frac{1}{3}$ of the remainder. He afterwards sold $\frac{1}{3}$ of what he had then left and there still remained 16 acres which he sold at $22.50 per acre. What was the whole farm worth at the same rate?

2. What is the interest of $450.90 from Dec. 7, 1895 to March 8, 1896 @ 6%?

3. If .05 of an acre is worth $1000, what would be the cost of a lot 34 feet by 100 at the same rate?

4. Find the value of $6.45 \times 8.25 \times 21 - 6.47 \times 8.25 \times 20$.

5. What is the depth of a cubical cistern that will hold 10000 standard gallons?

18.

1. A boy had a basket of apples. He gave $\frac{3}{8}$ of the whole to his brother and took 12 for himself. He gave what was left to his sister who found she had as many as the two others. How many were in the basket?

2. How many acres are there in a field 56.5 rods long, and 24.6 rods wide?

3. $400.00. St. John, Jan. 1, 1898.

Three months after date, for value received, I promise to pay James Hunter, or order, four hundred dollars.

Thomas Marks.

Discounted Jan. 8, at 6%. Find proceeds.

4. What is 107° C. on the Fahrenheit thermometer?

5. Change —31° F. to the centigrade reading.

19.

1. 3 men, 4 women or 5 boys can do a piece of work in $2\frac{1}{2}$ days. How long will it take a man, a woman, and a boy, working together to do it?

2. What must be the face of a note drawn May 4, at 3 months, so that if discounted on that date, at 6%, the proceeds will be $300?

3. What is the amount of $360 for 4 years at 5%, compound interest?

4. A sum of money amounts to $327 in 3 years at 3%, simple interest. What would it amount to at compound interest?

5. In what time will $640.80 amount to $760.90 at 6%, simple interest?

20.

1. At what time between twelve and one o'clock will the hour and minute hands be in a straight line?

2. Divide $3600 among A, B, and C, in the ratio of 5, 6, and 7.

3. A, B and C enter into partnership. A puts in $600 for 8 months, B $800 for 10 months, and C $1000 for 12 months. They gain $400. What is the dividend of each? (See Art. 44, Part III.)

4. $200.00. CHARLOTTETOWN, May 8, 1898.

Two months after date, for value received, I promise to pay Thomas Bancroft, or order, two hundred dollars, with interest at 6%. GEORGE CRANE.

Discounted May 9, at 7%. Find proceeds and bank discount.

5. At what rate, simple interest, will a sum of money treble itself in 40 years?

21.

1. The L. C. M. of 391 and another number is 12121, and the H. C. F. is 23. What is the other number?

2. If 3 horses are worth 10 cows, and 5 cows are worth 60 sheep, how many sheep are equal in value to 4 horses?

3. From the sum of $\frac{1\frac{1}{2}}{4}$ and $\frac{3}{4\frac{1}{2}}$ take the difference of their quotient and product.

4. Reduce $.692\dot{3}0\dot{7}$ to a common fraction.

5. A victualler buys a carcass of beef weighing 5 cwt. 94 lbs. at $7\frac{1}{4}$ cents per lb. He sells one-third of it at 28c. per kilogram, one-third at 22c. per kil. and one third at 16c. per kil. What was his gain?

22.

1. Find the sum of all the proper fractions, in their lowest terms, that can be formed having only one figure in the numerator and one in the denominator.

2. What is the cube of $\frac{21\frac{9}{11}}{29\frac{1}{11}}$?

3. What is the cube root of $\frac{63}{149\frac{1}{3}}$?

4. If 12 workmen, in 12 days, working 12 hours a day, can make up 75 yds. of cloth, $\frac{3}{4}$ yd. wide, into articles of clothing, how many yards, 1 yd. wide, can be made up into similar articles, by 10 men, working 8 hours a day, for 9 days?

5. A man who has a 2-acre field, twice as long as it is broad, can sell the whole at 10 cents per square foot, or he can sell it in lots of 100 ft. by 30 ft. for $380 each. Which is more advantageous, and how much? If he sells in building lots what fraction of the land will remain unsold?

23.

1. Find the difference between $\frac{2}{3}$ of $\frac{5}{8}$ of 6 shillings and .0875 of $\frac{2}{5}$ of £1.

2. What would it cost to dig a ditch 40 rods long, 3 feet wide, and $4\frac{1}{2}$ feet deep, at 18 cents a cubic yard?

3. A merchant bought a bbl. of apples for $3.20. The bbl. contained 400 apples, and two out of every 20 were decayed. At what rate per dozen must he sell the remainder so as to gain 25% on his outlay?

4. Of five legatees the first receives .3 of the bequest, the second .3, the third .03, the fourth .03 and the fifth the remainder amounting to $36400. How much does each of the others receive?

5. How much money must be put at interest at 6%, simple interest, to yield in interest $354.60 in 2 years 6 months?

24.

1. What is the weight of a gram in grains? How was the weight of the gram fixed?

Find the value of $\$5.67 \times \frac{3\frac{1}{7}}{7\frac{1}{5} \text{ of } \frac{3}{4}} \times \frac{\frac{5}{7} - \frac{1}{4}}{\frac{3}{8} + \frac{3}{7}}$

2. Divide .06321 by .0000003, and from 3.00828 take 1.0090106.

3. Find the difference between 3 miles and 2 mi. 7 fur. 39 rd. 5 yd. 2 ft 10 in.

4. When the income tax is 6d. in the £, a man pays £15 7s. 6d; what is his income?

5. A sum of money at simple interest has in $4\frac{1}{2}$ years amounted to $735, the rate of interest being 5%; what was the sum at first and in how many years more will it amount to $1140?

25.

1. There are in a light house three revolving lights. One revolves in 260 seconds, another in 195 seconds, and the third in 390 seconds. They are all in line with a small island out at sea at 11 o'clock. When will they be again in line?

2. A map is drawn on a scale of 10 miles to an inch, and a township is represented on it by a square whose side is half an inch. How many acres in the township?

3. At what price must a bookseller mark a book which cost $1.20 so that he may allow a discount of 10% to a student and still make 20%?

4. A square court-yard is bordered by a grand drive 10 yards wide, and the drive covers 4000 square yards. How many square yards in the enclosed grass-plot?

5. A commission merchant in Montreal sells 15460 lb. of wool, at $24\frac{1}{2}$ cents a pound. If his commission is 4% and exchange $\frac{1}{8}$ premium, how large a draft can he buy to send to his consignor in Charlottetown?

26.

1. A 3 per cent. stock is selling at 86. If a man who has $10000 to invest delays until it rises to 87, what change is made in his income?

2. A man who invested in 3% consols, received 4% on his investment after deducting an income tax of 1s. 4d. in the pound. What did he pay for the stock?

3. Which is the more advantageous, to buy flour at $5 a bbl. on 6 months credit, or at 4.87\frac{1}{2}$ cash, money being worth 7%?

4. A and B working together, can earn a sum of money in $8\frac{4}{7}$ days. B can earn it, working alone, in 20 days. In how many days can A earn it, if working alone?

5. A merchant buys 1000 lb. of tea at 25 cents a pound. He pays 20% duty and $27.80 for freight and other expenses. At how much a lb. must he sell the article to gain $33\frac{1}{3}$%.

27.

1. A man buys coal by the long ton and sells it at the same rate by the common ton. What per cent. does he gain?

2. A square field, one side of which is 200 yards, contains a circular pond one hundred yards across. How much dry land is there in the field?

3. If 1000 shingles cover 100 sq. ft., how many will it take to cover a barn 50 ft. long, 40 ft. wide, corner 21 ft. high, gable 36 ft. high, and rafter 25 ft. long?

4. A note of hand for $80, drawn May 1st, 1895, has the following endorsements:—June 10, '96, $20; Aug. 5th, '97, $15. What remains to be paid May 1st, 1898, interest being 6%?

5. What must be the depth of a cylindrical vessel, whose diameter (inside) is 3 ft., so that it may contain just 10 bushels?

28.

1. A, B, and C start from the same place at the same time, going in the same direction around an island whose circumference is $\frac{1}{2}$ mile. They travel at the rate of 3, 4,

and $4\frac{1}{2}$ miles respectively. In what time will they be together again, and how far will each have travelled?

2. A grocer gives 22 lb. of sugar for $1 cash, and 20 lb. on 1 year's credit. What is the rate of discount?

3. What sum must be invested in U. S. $4\frac{1}{2}\%$ bonds at 102, so as to provide an annual income of $1620?

4. Find the cost of excavating a cellar 6 ft. deep for a house 27 ft. by 31, at 20 cents a load for the first foot, 24 cents per load for the second, 28 cents a load for the third, etc.?

5. The cellar mentioned in the last question is walled 1 ft. above the ground with a wall $1\frac{1}{2}$ ft. thick, at $14 a perch. What is the mason's bill?

29.

1. How many cubic inches in the largest ball that can be cut out of a cubical block whose edge is 6 inches?

2. A man bought an article at 20% below the retail price. If he had paid a dollar more for it, it would still be 50 cents below the retail price. What was the retail price?

3. Bought 160 lb. of tea at 20 cents a lb., 140 at 30 cents, and 100 at 40 cents. At how much a lb. must I sell the mixture to gain 20%?

4. What principal will yield $60 interest in 219 days at $7\frac{1}{2}\%$?

30.

1. Find the sum of the series 2, 6, 18, etc., to 8 terms?

2. How many strokes does a clock which strikes the hours, strike in 13 weeks?

3. A grocer mixes 10 lbs. of tea at 40 cents per lb., 20 at 45 cents, and 30 at 50 cents, and sells the mixture at 70 cents per lb. How much does he gain per cent.?

4. If a hatter sells hats at $1.25 each and loses 25 per cent., what per cent. would he lose by selling them at $1.60 each?

5. What rate of interest does a person receive who invests in $4\frac{1}{2}$ per cent. stock at 90?

31.

1. The distance from Halifax to St. John by I.C.R. is 275 miles. A train leaves each place at 7 o'clock, the Halifax train running 25 miles an hour and the St. John train 30 miles. (*a*) When and where will they meet? (*b*) When will they be 40 miles apart? (*c*) If the St. John train leaves at 11.30, where will it meet the Halifax train?

2. Find the area of a circular bicycle track which measures 8 laps to the mile, measured on the smaller circumference, the track being 20 feet wide.

3. A man invested $2400 in a publishing business, and at the end of $2\frac{1}{2}$ years he withdrew $2940, being investment and profits. What annual rate of interest did his investment pay?

4. If a ton of coal occupies 40 cubic feet, what will it cost to fill a bin 12 ft. long, 6 ft. wide and 5 ft. deep, when coal is worth $3.90 a ton?

5. Iodine melts at 107° C., and boils at 178° C. At what temperature as indicated on a Fahrenheit thermometer does iodine melt and boil?

32.

1. A merchant vessel starts at 8 o'clock Monday from a point 45° north latitude, and sails due south at an average rate of 8 miles an hour. At 9 o'clock on Tuesday a cruiser starts in pursuit at the rate of 15 miles an hour. When and in what latitude will the vessel be overtaken.

2. Why do we subtract one hour from the time we use throughout the province to get railway time?

3. How many turns of a bicycle wheel 28 inches in diameter are made in going 10 miles?

4. A pond whose area is $\frac{1}{2}$ acre is covered with ice 18 inches thick. If 1 cubic foot of water weighs $62\frac{1}{2}$ lb., and sp. gravity of ice is .93, find the number of tons of ice on the pond.

5. A grocer buys a bbl. of sugar (gross weight 300 lb., tare 12 lb.) at $4\frac{3}{5}$ cents per lb. He sells $\frac{1}{4}$ of it at the

rate of 18 lb. for \$1. How much must he get per lb. for the rest so as to gain 20% by the transaction?

33.

1. Simplify $\dfrac{6+\dfrac{1}{6-\frac{1}{6}}}{4-\dfrac{1}{4-\frac{1}{4}}}+10\frac{8}{9}$.

2. Divide .0623 by .00002, and find the sum of $2.20\dot{3}+.65\dot{8}3+4.023\dot{5}7\dot{0}$.

3. If $\frac{3}{5}$ of an estate be worth \$7520, what is the value of $\frac{5}{8}$ of the estate.

4. A can do a piece of work in 27 days, and B in 15 days; A works alone at it for 12 days, B then works 5 days, and then C finishes the work in 4 days. In what time could C have done the work by himself.

5. \$1500. ST. JOHN, Jan. 1, 1897.

One year after date, for value received, I promise to pay John Smith, or order, fifteen hundred dollars, with interest, at 6%. JAMES JONES.

The following payments were made and endorsed on this note: March 16, \$100; June 13, \$400; Sept. 1, \$200. What was due Jan. 1, 1898, interest at 7%?

34.

1. A person sells 3% stock at 96 and invests the proceeds in 5% stock at par. How much per cent. is his income increased?

2. A owes B \$900, of which \$300 are due in 4 months, \$400 in 6 months, and \$200 in 9 months. What is the equated time for the payment of the whole amount?

3. What is the compound interest of \$2000, for 2 years, at 4 per cent., payable quarterly?

4. What reading on a Fahrenheit thermometer corresponds to 176° C.?

5. Find the sum of 32 terms of the series 1, $1\frac{1}{2}$, 2, etc.

35.

1. A father said to his son, who was 12 years old, "My age is equal to $\frac{5}{8}$ of the sum of your age and mine, with your age added." How old was the father?

2. A man paid \$54 for insurance on a house, at $3\frac{3}{4}$ per cent., and found that if the house was destroyed by fire he would recover its value, the premium paid and \$5 besides. What was the value of the property?

3. What is the difference between the simple and compound interest on £1800 for 3 years at 6 per cent?

4. A farmer sends his agent 415 barrels of potatoes, which the latter sells at \$1.52 a barrel, charging $2\frac{1}{2}\%$ commission. He invests the net proceeds in cloth at \$1.95 a yard, charging $3\frac{3}{4}\%$ commission. How many yards did the agent send the farmer?

5. Find the sum of the terms of the progression $\frac{4}{9}$, $\frac{7}{12}$, $\frac{13}{18}$, etc., to 63 terms.

36.

1. If 4 men, 5 women, 6 boys or 8 girls can do a piece of work in 47 days, how long will it take 2 men, 4 women, 5 boys and 8 girls working together to do it?

2. The discount on a sum of money at 5% is \$108, and the interest on the same sum at the same rate and time is \$125.28. Find the sum and the time.

3. 60 yards of carpet 27 inches wide are bought to cover a room 23 ft. 6 in. by 18 ft. The carpet cost 4s. 6d. per yd., and the remnant was sold at 3s. 4d. per yd. What was the cost of carpeting the room?

4. A bill due 4 mos. hence is discounted at 7% (true discount), and \$1267 is received for it. What is its face value? (Days of grace not reckoned).

5. Assuming a sovereign to be worth $\$4.86\frac{2}{3}$ in Canada and 25 francs 2 centimes in France, what will a traveller lose who changes £15 at the rate of 25 francs for £1?

37.

1. I lent a friend $1750, which he kept for 16 months. How long must he lend me $700 to return the favor?

2. I can sell my house for $4000 cash, or for $5000 payable in two years. If I accept the latter offer and receive its present worth in cash, at 10% discount, how much better off shall I be than if I accept the former offer?

3. Sum the progression $1\frac{7}{9}$, $2\frac{2}{3}$, 4, etc., to 10 terms.

4. How much stock at $105\frac{1}{2}$, brokerage $\frac{3}{4}\%$, can be purchased for $4781.25? What is the brokerage?

5. How many gallons in a rectangular cistern 16 ft. long, 10 ft. 6 in. broad, and 8 ft. 4 in. deep?

38.

1. Find the cube root of 5 to four decimal places.

2. If 30 men can dig a trench 108 ft. long, 8 ft. 9 in. wide, and 9 ft. deep, in $10\frac{4}{5}$ days of $6\frac{2}{3}$ hours each, how many days of eight hours each will it take 24 men to dig a trench 96 ft. long, 12 ft. 10 in. wide, and 12 ft. deep?

3. What must be the face of a note made on Jan. 9, at 3 months, in order that, if discounted at once, at 6%, the proceeds will be $200?

4. A merchant asked for goods $12\frac{1}{2}\%$ more than they cost him, but was obliged to take $12\frac{1}{2}$ less than he asked. What per cent. of the cost did he lose?

5. Three men bought a grindstone, 4 feet in diameter, paying equal sums. The first ground off his share; the second an equal share, and likewise the third. If one-fourth of the grindstone was left, what was the thickness ground by each?

39.

1. Ether boils at 95° F., and mercury boils at 662° F. Reduce these temperatures to the centigrade scale.

2. A merchant wishes to borrow $96.91 on a bill made on July 5th, for three months. What must be the face of the bill, interest being reckoned at $8\frac{1}{9}\%$?

3. What sum must a man invest in Dominion 6's at 101 in order to have a clear income of $1775.50, after paying an income tax of $1\frac{3}{4}$ cents on the dollar on all over $400?

4. A steamer going easterly in 0° latitude at the rate of 14 knots an hour, meets a ship in longitude 53° 25′ west, and 16 hours later meets another ship. In what longitude did the steamer meet the second vessel?

5. Calculate the specific gravity of alcohol from the following data:

Weight of flask empty = 14.326 grams.
" " " filled with water = 29.654 "
" " " " " alcohol = 26.741 "

40.

1. The population of a city increases $\frac{1}{15}$ each year, and its present population is 34560. What was its population two years ago?

2. A railroad runs through an estate for 18 miles, occupying a space 33 yds. wide, valued at $5.67 per acre. The owner receives compensation in land worth $12\frac{1}{2}$ cents per sq. rod. How many acres must he receive?

3. An agent received a consignment of wheat which he sold at a commission of 4 per cent., and invested the proceeds in sugar, less a commission of 5 per cent. His whole commission amounted to $\$107\frac{1}{7}$. What was the value of the sugar bought?

4. What must be the marked price of goods which cost $6 that the merchant may take off 10% and still make 25% profit?

5. A cubic foot of copper weighs 550 lb., and tin 462 lb. What is the weight of a cubic foot of a mixture of 5 parts copper and 3 parts tin?

41.

1. What is the diameter of a circular field which contains 20 acres?

2. When a florin is worth $19\frac{3}{4}$ cents, what must be

paid for a draft on Vienna which will pay a debt in that city of 800 florins?

3. When the days are of equal length, and it is noon in London (1st meridian), on what meridian is it then sunrise? sunset? midnight?

4. A box is made of plank $3\frac{1}{2}$ in. thick. Its dimensions on the outside are 4 ft. 9 in. by 3 ft. 7 in., and its height is 2 ft. 11 in. How many square ft. did it require to make the box and how many cubic feet will it hold?

5. A man who has a garden 100 ft. long and 80 ft. wide wishes to enclose it with a ditch, to be dug outside, 4 ft. wide. How deep must the ditch be dug so that the earth taken from it and placed on the garden may raise the surface 1 foot?

42.

1. A hired a house for 1 year at $300; at the end of 4 months he takes in B, and at the end of 8 months he takes in C. What rent, in equity, must each pay at the end of the year?

2. What fraction of an acre in a triangular field whose sides are 78, 84, and 90 yards?

3. 20% of $\frac{3}{8}$ of a number is what % of $\frac{3}{5}$ of it?

4. The highest common factor of two numbers is 23, and their least common multiple is 483. What is their product?

5. How many liters in 10 imperial gallons?

43.

1. A has 40 yards start of B. If B runs 7 yards for A's 5, how far must he travel to overtake A?

2. Find the missing term in the proportion, 17 : ( ) : : 19 : 95.

3. The sum of the sides of a triangle is 162 feet and they are in the rates of 13, 20, 21. Find the area of the triangle.

4. A broker charged me $1\frac{1}{3}\%$ for purchasing some bank notes that were selling at 25% discount, commission being charged on money invested. Three of the notes of $10 each and one of $50 became worthless. I sold the

remainder at par and gained $520. What was the face value of the notes purchased?

5. A farmer bequeathed £1200 to his three sons, leaving $\frac{1}{2}$ to his eldest son, $\frac{1}{3}$ to the next and $\frac{1}{4}$ to the third son. How much should each receive in order that the intention of the testator may be carried out? (Work by Proportion).

44.

1. When it was 1 o'clock a.m., on Jan. 1st, 1898, in Bangor, Me., 68° 47′ west, what was the time at New Mexico 99° 5′ west?

2. If a credit of 3 mos. be allowed each item in the following account, when might the whole be equitably settled by one payment?

SMITH FRASER.

1898.				TO DAVID SINGER.
Jan. 8,	To 30 bbls. fish,	@ $7.00	- - - - -	$420
Feb. 4,	" 90 yds. carpet,	" 1.50	- - - - -	135
Mar. 22,	" 300 bbls flour,	" 6.00	- - - - -	1800

3. For what sum must a note be drawn on July 3, at 3 months, so that discounted immediately at 7% it may produce $501.69?

4. If 3 per cent. stock be quoted at 90, how much must I invest in it so as to have an annual income of $2000, after paying a one per cent. income tax?

5. If the hands of a clock indicate 3 o'clock when the proper time is one minute to three, and 4 o'clock when the proper time is half a minute past 4, what is the proper time when the hands of this clock coincide between 4 and 5?

45.

1. What is the gain per cent. when 75% of the selling price is $\frac{1}{5}$ less than the cost?

2. $\frac{8}{19}$ of A's money equals $\frac{4}{9}$ of B's, and both together have $851. How much has each?

3. What can a man afford to pay for stock which yields a dividend of 12%, so that he may realize at least 8% on his investment?

4. What is the largest number of trees that can be set in a garden 120 yards square, so that the trees shall be at least 10 yds. apart and not less than 5 yds. from the fence by which the garden is enclosed?

5. A British shilling is 37 parts pure silver and 3 parts alloy, and 66 shillings weigh 1 lb. The United States dollar weighs 412.5 grains, and is 9 parts silver and 1 part alloy. How many shillings is a U. S. dollar worth?

46.

1. Find the compound interest of $500 for 15 years at 5%, the amount of $1 for the same time and at the same rate being $2.0789.

2. What is the amount or final value of an annuity of $300 for 5 years, at 4%, simple interest?

3. What sum will purchase an annuity of $240 for 3 years at 5%, compound interest? What is the final value?

4. A merchant dilutes a liquid, for which he paid $97\frac{1}{3}$ cents a gallon, with water, and sells the mixture at 73 cents a gallon. If he gain 20% on his outlay, how much water is there in every gallon sold?

5. An artisan received $3.75 a day and his board for his labor and paid $1.25 a day for his board when he was idle. At the end of 100 days he had saved $200. How many days did he work?

47.

1. A man hired with a farmer for $1.25 a day and his board, for every day he worked. On idle days he paid 50 cents for his board. At the end of 100 days he received $90. How many days did he work?

2. If I purchase bank stock at 28 per cent. premium, and the bank pays a dividend of 9%, what interest do I receive on my investment?

3. Which would be the more advantageous way for the Dominion to borrow money: on bonds bearing interest

at $2\frac{1}{2}\%$, and selling at 91, or on bonds bearing interest at 3% and selling at 101?

4. The true discount on £566 10s. for 9 months is £16 10s., what is the rate?

5. Find the cube root of .0001 to 4 places of decimals.

48.

1. A farmer wishes to fence in a part of his pasture, to contain 225 square rods, as a field on which to raise oats. Allowing himself $1.50 per day for his labor, fencing costs him 40 cents a rod. He marks out a field 25 rd. long and 9 rd. wide. How much less would it cost to fence a square field of the same area?

2. About how many miles must a man walk to plough an acre, turning a furrow of 9 inches?

3. What is the length of a rectangular field, 80 yds. wide, which contains 2 acres? Of a field 100 yds. wide which contains 5 acres.

4. A merchant's assets are $3932.50 and his liabilities $4840. How much can he pay on the dollar.

5. In an English university boat race the Cambridge crew rowed 39 strokes per minute and the Oxford crew 41 strokes per minute; but 19 strokes of Cambridge were equal to 20 of Oxford. The Cambridge crew rowed over the course of 4 miles in 25 minutes. Find the number of feet and the number of seconds by which the race was won.

49.

1. How many square feet of iron plate, $\frac{1}{4}$ of an inch thick, can be made from a cylindrical shaft 20 ft. long and 4 inches in diameter?

2. How long a trip up a river and back again to the starting point can be made in 6 hours by a small steamer which can travel 10 miles an hour with the current and 5 miles an hour against it? Find also the velocity of the current.

3. A, B and C were partners in business. A's capital was $\frac{2}{3}$ of B's, and B's was $\frac{4}{7}$ of C's. A's capital was in the

business 8 months, B's 9 months, and C's 10 months, and their net gain was $5348. What was the share of each?

4. A man, whose horse is tethered by a halter to a stake in a grass field, uses a rope 30 ft. long. He wishes the horse to have one-third of the fresh grass which the full length of the rope would give him in the forenoon, one-third in the afternoon and the remaining third at night. What length of rope must he allow the horse in the forenoon and in the afternoon?

5. Thomas Forbes gave Hugh Miller on March 10, 1898, mdse. worth $75; April 20, mdse. on 2 mo. $120; and May 30, on 3 mo. mdse. $130. Hugh miller paid Thos. Forbes on March 27, $60 cash; on April 15, a 10-day draft for $100, and on June 4, a 30-day draft for $50. When should the balance be paid?

50.

1. The first term of a geometrical progression is $4\frac{4}{5}$, the last $\frac{8}{405}$, and the ratio $\frac{1}{3}$; what is the sum of the series?

2. A watchmaker sold 20 watches that cost him $8 a piece on the following plan: He secured 20 persons who contributed $1 each, and at the end of a week one of them drew a watch by lot and retired. The remaining 19 contributed $1 each and at the end of the second week one drew a watch by lot and retired, and so on until every person received a watch. What profit did the watchmaker reap.

3. A and B enter into partnership. A's capital is to B's as 5 to 8; at the end of 4 months A withdraws $\frac{1}{2}$ of his capital, and B $\frac{2}{3}$ of his; their gain, at the end of a year is $4000; how should this gain be divided?

4. Stock bought at a discount of 30%, yields a dividend of 4% every six months. What is the annual rate per cent. on the investment?

5. A cargo was insured $4\frac{3}{8}$%, to cover $\frac{5}{8}$ of its value The premium was $122.50; what was the value of the cargo?

51.

1. If 7 men *or* 12 boys hoe a field of potatoes in 6 days, working 10 hours a day, in how many days, of 8 hours each, can 14 men *and* 6 boys hoe a field five times as large?

2. A grocer was selling cans of peaches so as to gain 20%. He increased the price to $1.50 per dozen cans and his profit rose to $33\frac{1}{3}\%$. At what rate per doz. was he selling them at first?

3. A fruiterer buys equal quantities of apples at 2 for a cent and 3 for a cent, and mixes them. A customer comes and asks for 75 cents worth. How many can the dealer give him, selling at a gain of 25%?

4. The stock in trade of two partners in a tea business consisted of 1020 chests of tea. B's share in the business being $\frac{8}{9}$ of A's. Having agreed to dissolve partnership, B took 900 chests, A took 120 and received $1050 from B. Tea then rising $16\frac{2}{3}\%$ in price, each sold his stock. After the sale, what fraction is A's property from this source, of B's?

5. A square field is surrounded by a wall. The part immediately within the wall, all round the field, is covered with gravel, and is 13 ft. in width, and two straight gravel walks, 13 ft. in width, join the middle parts of the opposite sides. The ungravelled part of the field contains $2\frac{1}{2}$ acres. What is the length (in yards) of the bounding wall?

52.

1. The edges of a rectangular solid, whose content is 64 cubic feet, are in the ratio of 1 : 2 : 4. What are the lengths of the edges?

2. A man hired a team to drive from Long Point Bridge to Port Hood, a distance of 15 miles, and back again, for $3, with the privilege of taking in 1 or 2 persons at any place on the road. When 4 miles from Long Point Bridge, he took up the miller, and when 10 miles from Long Point Bridge he took up the postmaster. On his return he set them down at the points at which he took them up. How much, in equity, should each pay?

3. At what times between 22 and 23 o'clock are the hands of a watch at right angles, the hour hand making but one revolution of the dial plate in 24 hours, and the minute hand making a revolution every hour?

4. A ten-foot pole casts a shadow of 15 feet. What is the height of a tree that casts a shadow of 80 feet?

5. What sum will purchase an annuity of $500 for 7 years, at 6%, compound interest, the amount of $1 for 7 years at 6 per cent., compound interest, being $1.50363.

53.

1. A railway train which runs at the rate of 20 miles an hour leaves a station 50 minutes before another train going in the same direction and travelling at the rate of 30 miles an hour. In what time will the second overtake the first?

2. How many rolls of paper (the roll is usually 7 yds. long and 18 in. wide), must be purchased for a room 18 ft. long, 13 ft. wide and 10 ft. high? There are 18 in. lost in "matching" each strip with the next, but that is to be considered as off-set by the doors and windows.

3. At 90 cents per cubic yard what will it cost to build a brick and cement wall 72 ft. long, 7 ft. high, 3 ft. thick at the base and 18 in. at the top?

4. A cubic foot of air weighs about .0808 lb. and a cubic foot of hydrogen about .0056 lb. A spherical balloon, the diameter of which is 20 ft., is filled with hydrogen gas. What weight will it support?

5. A ship sails due north for 3 hours at the rate of 7 knots an hour, and then due east for 4 hours at the rate of 8 knots an hour. How many knots is she then from her starting point.

54.

1. Divide $4941 among A, B and C, so that $3\frac{1}{2}\%$ of A's share, $3\frac{3}{4}\%$ of B's share, and $4\frac{1}{2}$ of C's share may all be equal.

2. The excess of the present worth of a sum of money

due in 1 year, interest 5%, over the present worth when interest is reckoned at 6% is \$0.50. Find the sum.

3. A, B and C enter into a partnership. A and B put in \$8475; A and C \$8650; B and C \$8875. They gain \$2062.50. What is each one's share in the profits?

4. A merchant bought 200 meters of cloth in France at $16\frac{1}{4}$ francs per meter, and after paying freight and duty amounting to $12\frac{1}{2}$ cents per yard, he sold it in Halifax at \$4.62$\frac{1}{2}$ per yd. What was his gain?

5. If 4 men mow 15 acres in 5 days of 14 hours each, in how many days of 13 hours each can 7 men mow $19\frac{1}{2}$ acres?

55.

1. What per cent. is gained by buying stock at a discount of 20% selling it at a discount of 10%?

2. If the true discount on a bill due 8 mos. hence (days of grace not reckoned), at $7\frac{1}{2}$ is \$97.50, what is the amount of the bill?

3. A man invested \$28700 in U. S. 6's at $107\frac{1}{2}$, the brokerage being $\frac{1}{8}$%. What will be his clear income after an income tax of 5% has been deducted?

4. A bill of £90 10s. is due in Edinburg, Scotland, by a school section in Canada for apparatus imported. What will the secretary of trustees pay for a bill of exchange to liquidate this debt when sterling exchange is quoted at $109\frac{3}{4}$?

5. A grain of gold can be beaten into a leaf 8 in. square. How many of these leaves laid one upon another would make a block 1 in. thick, the weight of a cubic inch of gold being 10 ounces.

56.

1. A man having invested \$10000 at 4%, payable half-yearly, wishes to receive his interest in equal portions payable monthly and in advance. How much should he receive every month?

2. A and B hired a horse and wagon for 30 days for

$30. A alone used it 10 days and B, 5 days. They used it together the remainder of the time. How much should each pay?

3. A man invests a certain sum at 5% and twice the sum at 6%. His income from both investments amounts to $1700. How much has he invested at each rate?

4. A man who owes a merchant a debt payable in 7 months agrees to pay one-half cash and three-eighths in 6 months. What time should he be allowed to pay the balance?

5. The head of a fish is 12 inches long, the tail is as long as the head and half the body, and the body is as long as the head and tail together; what is the length of the head and tail?

57.

1. A merchant in Ontario sends flour to Prince Edward Island to be sold on commission at 5%. His agent is to invest the proceeds in herring after deducting his commission at 3%. How much will be invested in herring?

2. What is the sum of the numbers from 1 to 90 inclusive?

3. A lad bought a lot of oranges at $2\frac{1}{2}$ cents a piece and twice as many at 2 cents a piece. He sold them all at 33 cents a dozen and made $4.20 profit. How many did he buy?

4. A cubical box is made to just hold a globe. If the depth of the box is 15 inches, what is the solidity of the globe?

5. Work by proportion: If 17 men in 24 days earn $612, in how many days will 68 men earn $\frac{1}{5}$ that sum?

58.

1. If the population of a country which is 450523 in 1891 increases annually 10 per cent. what would the population be in 1901?

2. A ladder 52 ft. long stands close against the side

of a building. How many feet must it be taken out at the bottom, that the top may be lowered 4 ft.

3. A man purchases goods as follows: June 4, 1897, $240.75; Aug. 9, 1897, $137.25; Aug. 29, 1897, $65.64; Sept. 4, 1897, $230.36; Nov. 12, 1897, $36. If all the goods are bought on a credit of 6 mos., on what date may the bill be settled by paying the amount of the several purchases?

4. A's money is 28% more than B's. What per cent. is B's less than A's?

5. Find the difference in the incomes obtained by investing $7700 in $2\frac{3}{4}$ per cent. consols at $96\frac{1}{4}$, or an equal sum in 4 per cent. government stock at 110.

59.

1. A man invests $1761.75 in 3 per cents at $97\frac{3}{4}$, and sells out when they have risen to par. What sum does he clear, reckoning brokerage $\frac{1}{8}$% both in buying and selling?

2. Extract the square root of 1191078144.

3. Extract the cube root of 94818816.

4. What is the value in sterling money of 294.32 francs when the course of exchange is 24.90 francs per pound sterling?

5. The distance from X to Z is 170 miles. A train, travelling at the rate of 40 miles an hour leaves X at 10 o'clock, and at the same time another train leaves Z travelling at the rate of 50 miles an hour. If the latter train stops from 11 to 11.30 and then starts again at the same rate as before, when and where will they meet?

ANSWERS.

Exercise I.—(Page 7).—**1.** 6. **2.** $10\frac{11}{14}$. **3.** $25\frac{3}{10}$. **4.** $16\frac{1}{21}$. **5.** $1\frac{31}{171}$. **6.** $1\frac{18}{35}$. **7.** $4\frac{917}{2376}$. **8.** $24\frac{115}{144}$. **9.** $12\frac{7}{33}$. **10.** $3\frac{47}{126}$. **11.** $15\frac{9}{10}$. **12.** $\frac{6}{35}$. **13.** $2\frac{124}{133}$. **14.** $5\frac{5}{19}$. **15.** $\frac{21}{64}$. **16.** $\frac{2}{37}$. **17.** $\frac{129}{377}$. **18.** $\frac{45}{88}$. **19.** $\frac{1}{13}$. **20.** $\frac{252}{1005}$. **21.** $1\frac{997}{1119}$. **22.** $25\frac{61}{72}$. **23.** $16\frac{33}{67}$. **24.** $3\frac{7}{80}$. **25.** $3\frac{2241}{2414}$. **26.** 6. **27.** $68\frac{11}{22}$. **28.** $1\frac{5}{11}$. **29.** 2. **30.** $\frac{1}{3}$.

Exercise II.—(Page 9).—

Exercise III.—(Page 10).—**1.** $531\frac{1}{4}$. **2.** $29\frac{1331}{10890}$. **3.** .093567. **4.** 2. **5.** .992424. **6.** $366916\frac{2}{3}$. **7.** $2\frac{1}{3}$. **8.** 3.2034. **9.** 37 mi. 4 fur. 1 ft. $6\frac{3}{4}$ in. **10.** 13s. $2\frac{1}{3}$d. **11.** 180.7+. **12.** 1.7142. **13.** 11910.646 yd. **14.** 436340.52 sq. in. **15.** $\frac{1}{2}$.

Exercise IV.—(Page 12).—**1.** $.\dot{6}$, $.1\dot{6}$, $.8\dot{3}$.

2. $.\dot{7}14285\dot{7}$, $.6428571\dot{4}$, $.3\dot{1}8\dot{2}$.

3. $.0\dot{1}42857\dot{1}$, $.14583\dot{3}$, $.\dot{1}$.

4. $3.0\dot{0}\dot{1}$, $200.0\dot{1}2345679\dot{1}$, $.5568\dot{1}\dot{0}$.

Exercise V.—(Page 13).—**1.** $\frac{8}{9}$, $\frac{5}{11}$, $\frac{6}{101}$. **2.** $\frac{334}{999}$, $\frac{6}{1111}$, $\frac{111}{333333}$. **3.** $6\frac{55}{101}$, $14\frac{16}{999}$, $100\frac{59}{999}$. **4.** $1\frac{88}{9999}$, $18\frac{1}{999}$, $60\frac{1}{3333}$. **5.** $6\frac{29}{333}$, $\frac{40}{99999}$, $\frac{16}{999}$. **6.** $8\frac{8}{9999}$, $\frac{37732}{370737}$, 1.

Exercise VI.—(Page 14).—**1.** $\frac{11}{90}$, $\frac{24}{45}$, $\frac{11}{225}$. **2.** $\frac{8}{126}$, $\frac{837}{9999}$, $\frac{6313}{99990}$. **3.** $\frac{1013}{30000}$, $\frac{79}{99000}$, $\frac{79}{99000}$. **4.** $\frac{4}{10989}$, $\frac{2}{2475}$. $\frac{1}{1782}$. **5.** $6\frac{234001}{9999000}$, $17\frac{592}{33000}$, $64\frac{1}{90}$. **6.** $4\frac{1}{99000}$, $4\frac{1}{99000}$, $29\frac{87311}{999990}$.

Exercise VII.—(Page 17).—**1.** $14.\dot{7}69587\dot{1}$. **2.** $2.\dot{8}96178807069\dot{3}$. **3.** $53.8198\dot{6}38274\dot{1}$. **4.** $52.52\dot{6}228203901471\dot{3}$. **5.** $17.809250213\dot{8}$. **6.** $339.625\dot{2}68352\dot{1}$.

Exercise VIII.—(Page 18).—**1.** $71.8\dot{6}19\dot{3}$. **2.** $291.552\dot{4}$. **3.** $13.8182\dot{4}$. **4.** $44.7\dot{8}9$. **5.** $500.91\dot{6}$. **6.** $3.904\dot{6}$. **7.** 1218.6. **8.** $.6\dot{1}3\dot{6}4073\dot{1}$. **9.** $3451.3\dot{8}6$. **10.** 0.

Exercise IX.—(Page 19).—**1.** $.0\dot{3}$. **2.** $9.92\dot{8}$. **3.** $.0\dot{8}2$. **4.** $1.\dot{8}$. **5.** $389.\dot{1}8\dot{5}$, **6.** $778.1\dot{4}8$. **7.** $750730.5\dot{1}8$. **8.** $31.79\dot{1}$. **9.** 34998.4199003+. **10.** $2.2\dot{9}7$.

Exercise X.—(Page 19).—**1.** 1.31034+. **2.** $55.\dot{6}\dot{9}$. **3.** $4.87\dot{3}1707\dot{}$.
4. $7.7\dot{2}\dot{}$. **5.** 2. **6.** $1.\dot{1}$. **7.** $1890.3\dot{0}\dot{3}$. **8.** $.0001\dot{3}$. **9.** $.0\dot{6}$. **10.** $.1\dot{0}\dot{9}$.

Exercise XI.—(Page 23).—**1.** 23. **2.** 29. **3.** 35. **4.** 47. **5.** 46. **6.** 11. **7.** 51. **8.** 67. **9.** 75. **10.** 89. **11.** 93. **12.** 77.

Exercise XII.—(Page 25).—**1.** 127. **2.** 253. **3.** 309. **4.** 408. **5.** 599. **6.** 608. **7.** 647. **8.** 807. **9.** 907. **10.** 897. **11.** 985. **12.** 4968. **13.** 3973. **14.** 1025. **15.** 2187.

Exercise XIII.—(Page 26).—**1.** 1.442. **2.** .669. **3.** .310. **4.** $\frac{4}{7}$ or .571. **5.** $\frac{5}{7}$ or .714. **6.** .480. **7.** 4.973. **8.** 7.640. **9.** 9.654. **10.** .497. **11.** 1.072. **12.** 1.912. **13.** 3.666. **14.** .046. **15.** 3.932. **16.** 5.561.

Examination Paper No. I.—(Page 27).—**1.** $3.59+. **2.** .2070. **3.** .2828. **4.** $25\frac{25}{33}$. **5.** £85 11s. $10\frac{1}{11}$d.

Examination Paper No. II.—(Page 27).—**1.** $\$1\frac{37}{100}$. **2.** 900 rd. **3.** .1082. **4.** 4444.65 lb. **5.** 11.

Examination Paper No. III.—(Page 27).—**1.** $82\frac{1}{7}\%$. **2.** $109\frac{1}{7}\%$. **3.** $2.12\dot{2}20766\dot{}$. **4.** 6s. 8d. **5.** $3.06.

Examination Paper No. IV.—(Page 27).—**1.** $1800. **2.** 41.2 ft. **3.** $44.765. **4.** 2200 acres. **5.** 2.55+.

Examination Paper No. V.—(Page 28).—**1.** 4.1888 cu. ft. **2.** .8796. **3.** 136.8 sq. ft. **4.** 1.

Examination Paper No. VI.—(Page 28).—**1.** $62\frac{1}{2}\%$. **2.** $.6\dot{4}59368\dot{}$. **3.** 20 sq. rd. **4.** 31c. **5.** 1 ft. 1 in.

Exercise XIV.—(Page 32).—**1.** 13112, 15786, 14620. **2.** 114241, 44132, 2042022. **3.** 9832, 126683, 143806. **4.** 3950, 6319, 52033. **5.** 7e58, 39619, 35013. **6.** 311522, 2214501, 11143$\frac{1}{2}$123. **7.** 2766, 22875. **8.** 47055, 29538, 21. **9.** 13226, 39148, 578. **10.** 15916, 76295, 53123.

Exercise XV.—(Page 32).—**1.** 154143_{6}. **2.** 204428_{9}. **3.** 204414_{8}. **4.** $2t7715_{11}$. **5.** 5372702_{8}. **6.** 473126656_{9}. **7.** $51234t7_{11}$. **8.** 4124302_{6}. **9.** $2146ee9_{12}$. **10.** 33226_{7}.

Exercise XVI.—(Page 42).—**1.** £72 6s. $7\frac{1}{2}$d. **2.** 124 lbs. 8 oz. 16 grs. **3.** 124 lbs. 8 oz. 16 grs. **4.** 5 t. 19 cwt. 90 lbs. 6 oz. 8 dr. **5.** 108 mi. 4 fur. 2 rd. 4 yd. 3 in. **6.** 1098 bu. 1 pk. 1 gal. 2 qrt. 1 pt. **7.** 8 wk. 3 da. 1 hr. 44 min. 36 sec. **8.** 7 Hm. 8 Dm. 9 m. 2 dm. 4 cm. 7 mm. **9.** 5 cu. yd. 7 cu. ft. 702 cu. in.

10. 5 sq. rd. 1 sq. yd. 3 sq. ft. 67 sq. in. **11.** 44010 far. **12.** 767904240 sec.
13. 403710 cm. **14.** 752112 dr. **15.** 1214 pt. **16.** 40371000000 qcm.
17. 1865106 in. **18.** 120166 gr. **19.** 120166 gr. **20.** 15250032 sq. in.

Exercise XVII.—(Page 43.—**1.** 4 T. 7 cwt. 72 lb. 2 oz. 12 dr.
2. 4180 mi. 1 fur. 13 rd. 2 yd. 2 ft. 6 in. **3.** 357 A. 27 sq. rd. 26 sq. yd. 8 sq. ft. $107\frac{1}{2}\frac{3}{3}$.
4. 35101 ; £24 0s. $10\frac{3}{4}$d. **5.** 297 lb. 8 oz. 18 dwt. 8 gr.; 3298 lb. 3 oz. 3 dwt. 8 gr.
6. 1 lb. 2 oz.; 3 cwt. 19 lb. 8 oz. **7.** 3£ 12s. $2\frac{3}{4}$d.; 14 farthings.
8. 108 ; No remainder. **9.** 4 cwt. 87 lb. $7\frac{3}{7}\frac{2}{3}$ dr.
10. 1 mi. 7 fur. 23 rd. 3 yd. 2 in. **11.** 10 A. 65 sq. rd. 16 sq. yd. 4 sq. ft. 136 sq. in.
12. 2379980 min. **13.** 4[illegible] 7 fur. 9 rd. 2 yd. 2 ft. $7\frac{5}{7}$ in. **14.** $12\frac{8}{2}\frac{3}{5}\frac{9}{5}\frac{6}{1}\frac{1}{4}\frac{3}{4}\frac{7}{3}\frac{7}{6}$.
15. 9 cwt. 42 lb. 10 oz. [illegible]140. **17.** The steamer by 98 yds.
18. 5 ft. 3 in. **19.** 2 lb. 5 oz. 2 scr. $15\frac{1}{1}\frac{2}{8}\frac{4}{7}$ grs. **20.** 1 cwt. 12 lb. 4 oz.

Exercise XVIII.—(Page 44).—**1.** .282. **2.** 15 cwt. 56 lb. 4 oz.
3. .872595+. **4.** 6 oz. 5 dr. 19.68 gr. **5.** $\frac{3}{2}\frac{5}{7}$. **6.** £2 5s. $8\frac{8}{4}\frac{8}{5}$d.
7. 4 yd. 2 ft. 2.566 in. **8.** .09027̇. **9.** $15.74. **10.** $4.07.
11. 80516.4 in. **12.** 3884. **13.** $16.50.
14. 107 sq. rd. 2 sq. yd. 6 sq. ft. 51.6 sq. in. **15.** 7.164116̇ days.

Exercise XIX.—(Page 45).—**1.** 163.5^{m} . . . . .
2. 427 yd. 1 ft. 10 in. **3.** 250^{m}. . . . . **4.** 729.24+.
5. 89.6^{Km}. **6.** 945^{l}. **7.** 100 gal.
8. 312 bu. 3 pk. 1 gal. **9.** $204.75. **10.** $55.02.
11. $.299. **12.** 56.764 lb. **13.** 855^{g} . . . . . **14.** 1200^{g} . . . .
15. 16 T. 10 cwt. 69 lb. **16.** 1.200^{g}. **17.** 132.05 lb.
18. $187.20. **19.** 2 lb. 9 oz. 15 dwt. 19.032 gr. **20.** 1.4^{m}.

Exercise XX.—(Page 46).—**1.** $133.022. **2.** £175 3s. $10\frac{1}{2}\frac{1}{0}$d.
3. 500 marks. **4.** $180.0708. **5.** $837\frac{1}{2}$ drachmas. **6.** $.2261.
7. $2059.7115. **8.** 22321.428 florins. **9.** 9951.44 marks.
10. $744.98. **11.** $744.98. **12.** $744.98. **13.** 2000 francs in each case.
14. 5000 crowns in each case. **15.** $105\frac{8}{1}\frac{1}{0}$ pfennigs, $74\frac{1}{4}\frac{7}{2}$ kreutzers.
16. 3 rupees, $1\frac{7}{1}\frac{7}{2}\frac{3}{3}$ annas. **17.** 49 roubles 20 copecks.
18. £39 13s. $1\frac{3}{4}$d. **19.** 1357 marks, 14 pfennigs.
20. 186.56 ore, 463 reis.

Exercise XXI.—(Page 48).—**1.** 45 min. 38 sec. past 7 o'clock. **2.** 1 hr. 3 min. 2 sec. **3.** 20 min.; 53 min. 22 sec. **4.** 1 hr. 38 min. 4 sec. **5.** 20° 35′. **6.** 26° 15′. **7.** 14 min. 22 sec. faster. **8.** 73° 21′. **9.** 25 min. 34 sec. past 16 o'clock. **10.** 4 hr. 12 min. 41 sec. **11.** 51 min. 56 sec. past 6 o'clock. **12.** 5 hr. 58 min. 51 sec. **13.** East; 1425 geographical miles. **14.** 2 hr 20 min. 49 sec. **15.** 3 hr. 13 min. $38\frac{2}{3}$ sec. **16.** 33° 51′ 45″

Examination Paper No. VII.—(Page 49).— **1.** 24 min. past 17 o'clock. **2.** 4 h. 20 m. **3.** .237083. **4.** 576000 T. **5.** $110.05.

Exercise XXII.—(Page 52).—**1.** 149°F. **2.** 30°C. **3.** 68°F. **4.** $78\frac{1}{3}$°C. **5.** 110°C, $167\frac{7}{9}$°C. **6.** 104°, 122°, 140°, 158°, 176°, 194°, 212°F **7.** 45°, 55°, 60°, 80°, 95°C. **8.** 10°C. or 18°F. **9.** 77°F. **10.** $4\frac{4}{9}$°C. **11.** 8°, 52°, 63°, 91°C. **12.** $-38\frac{1}{9}$°F. **13.** $537\frac{7}{9}$°, $587\frac{7}{9}$°, $676\frac{4}{9}$°, $898\frac{4}{9}$°C. **14.** Equal.

Examination Paper No. VIII.—(Page 53).—**1.** $10924\frac{1}{2}\frac{1}{3}$. **2.** $48\frac{67}{336}$ kreutzers, or about 16 cents. **3.** $33\frac{1}{3}$ miles. **4.** 73.3 ft. **5.** 1.2345679.

Exercise XXIII.—(Page 53).—**1.** $0. **2.** $2100. **3.** 150. **4.** $21\frac{19}{36}$%. **5.** $9\frac{13}{36}$% gain. **6.** $88\frac{8}{9}$%. **7.** $11\frac{1}{9}$%. **8.** $12\frac{1}{2}$%. **9.** 19%. **10.** $66\frac{2}{3}$%, $29\frac{17}{17}$%. **11.** $52\frac{1}{2}$%. **12.** $8\frac{1}{3}$%. **13.** $2\frac{2}{7}$%. **14.** $12\frac{1}{2}$%, $14\frac{2}{7}$%. **15.** The latter by $19.50. **16.** $46\frac{2}{7}$%. **17.** $3.98. **18.** $150. **19.** $3200. **20.** $32\frac{1}{8}$.

Exercise XXIV.—(Page 55).—**1.** $54.09. **2.** $50.69. **3.** $36\frac{9}{10}$% nearly. **4.** $143.28. **5.** $1.25%. **6.** $22.31. **7.** $2211.20. **8.** $24.83. **9.** $44.78. **10.** 500 bbls. **11.** $5.46. **12.** $67.50. **13.** $9000. **14.** $289.40. **15.** $4397.25. **16.** $4400. **17.** $2500. **18.** $4300. **19.** $900. **20.** $17\frac{3}{37}$%.

Exercise XXV.—(Page 57).—**1.** $1.08. **2.** $341.55. **3.** $380.10 **4.** $7\frac{1}{2}$%. **5.** $90.68. **6.** $1273.30. **7.** $1628.15. **8.** $2493.75. **9.** $434.25. **10.** $969.75.

Examination Paper No. IX.—(Page 58).—**1.** $47\frac{7}{19}$%. **2.** $2029.57. **3.** $24\frac{1}{10}$%. **4.** 625. **5.** $15.603.

Exercise XXVI.—(Page 59).—**1.** $117. **2.** $153.23. **3.** $159.60. **4.** $136.71. **5.** $52.585. **6.** $3.95. **7.** $287.93. **8.** $89.28+. **9.** $66.855. **10.** $91.97. **11.** $467.15. **12.** $69.55. **13.** $46.715. **14.** $14.43. **15.** $238.38. **16.** $392.14. **17.** $4898.19.

18. $490.22. **19.** $2249.10. **20.** $550.112. **21.** 7%. **22.** 5%.
23. $4\frac{3}{4}$%. **24.** 1 yr. 135 days. **25.** $1\frac{1}{4}$ years.
26. 1 yr. 216 days. **27.** $9250. **28.** $927.10. **29.** $550. **30.** $500.

Examination Paper No. X.—(Page 60).—**1.** 24 hr. 42 min.
2. $1264.50. **3.** $92. **4.** $580. **5.** $33.

Exercise XXVIa.—(Page 62).—**1.** $113.31. **2.** $79.67.
3. $104.59. **4.** $282.36. **5.** $233.36.

Exercise XXVII.—(Page 63).—**1.** $6.15, $843.85
2. $5.42, $744.58. **3.** $1.85, $498.15. **4.** $.74, $74.26.
5. $5.07, $1254.93. **6.** $30.35, $5399.65. **7.** $31.24, $5398.76.
8. $3.79, $481.21. **9.** $3.92, $481.08. **10.** $3.92, $481.08.

Exercise XXVIII.—(Page 64).—**1.** $750, $75. **2.** $900, $162.
3. $315, $37.80. **4.** $1255, $263.55. **5.** $9250, $1618.75.
6. $72.85, $18.03. **7.** $925, $74. **8.** $10,000, $49.31.
9. $500, $78.82. **10.** $699.44; $64.84. **11.** $.54.
12. The latter. **13.** $2640. **14.** $1680. **15.** $824.

Examination Paper No. XI.—(Page 65).—**1.** $15\frac{5}{11}$ yr. **2.** $500.
3. $98; $\frac{3}{16}$; $\frac{3}{100}$ **4.** 20%. **5.** 15 yr. 1 mo. 20 da.

Exercise XXIX.—(Page 69).—**1.** $2531.25. **2.** $16912.50.
3. $207.90. **4.** $1000. **5.** $4000. **6.** 50 shares. **7.** $1071.43.
8. $573.94. **9.** $892.51. **10.** No gain or loss. **11.** The latter.
12. The former. **13.** $62\frac{1}{2}$%. **14.** $80. **15.** $5967.50. **16.** $3788.80.
17. $27309.38. **18.** $220. **19.** The latter. **20.** $\frac{33}{175}$. **21.** $76\frac{13}{17}$.
22. $217. **23.** $14400. **24.** $6936. **25.** $2214.50

Examination Paper No. XII.—(Page 70).—**1.** 20 gal. **2.** $50.
3. $945.47 gain. **4.** $17.50, $482.50. **5.** $30\frac{3}{32}$ cents a lb.

Exercise XXX.—(Page 73).—**1.** $850.85. **2.** $1556.10.
3. $873.59. **4.** $210.93. **5.** 4853.93. **6.** $642.40. **7.** $3540.94.
8. $997.22. **9.** $750.62. **10.** $4926.11. **11.** $1992.53. **12.** $917.01.

Exercise XXXI.—(Page 77).—**1.** $1727.30.
2. $4.82\frac{1}{2}$ for each £1. **3.** $1523.19. **4.** $682.50. **5.** $1418.55.
6. $1557.09. **7.** $1539.17. **8.** $204.58. **9.** $1998.36. **10.** $4.8665.

Exercise XXXII.—(Page 79).—**1.** 2 mos. 18 da. **2.** 41 da.
3. $3\frac{1}{4}$ mo. **4.** June 29th. **5.** Aug. 19th. **6.** June 6th. **7.** $375.
8. $1058.04; equated time Sept 23.

Exercise XXXIII.—(Page 83).—**1.** Nov. 29th, 1897. **2.** July 6th, 1898. **3.** March 11th of the same year. **4.** $6\frac{1}{2}$ mos. **5.** June 15th, 1896.

Exercise XXXIV.—(Page 84).—**1.** $\frac{1}{8}$. **2.** $8\frac{1}{7}$ hr. **3.** $6\frac{1}{4}$ da. **4.** 20. **5.** 3 da. **6.** $9\frac{3}{13}$. **7.** $4\frac{4}{11}$ da. **8.** $7\frac{3}{17}$. **9.** $24\frac{1}{2}$. **10.** 13.

Exercise XXXV.—(Page 86).—**1.** $16\frac{4}{11}$ min. past 3. **2.** $32\frac{8}{11}$ min. past 6. **3.** $27\frac{3}{11}$ min. past 5. **4.** $49\frac{1}{11}$ min. past 9. **5.** $43\frac{7}{11}$ min. past 8. **6.** $54\frac{6}{11}$ min. past 10. **7.** $16\frac{4}{11}$ min. and $49\frac{1}{11}$ min. past 6. **8.** $21\frac{9}{11}$ min. and $54\frac{6}{11}$ min. past 7. **9.** $10\frac{10}{11}$ min. and $43\frac{7}{11}$ min. past 11. **10.** $38\frac{2}{11}$ min. past 1. **11.** 6 o'clock. **12.** $10\frac{10}{11}$ min. past 8.

Exercise XXXVI.—(Page 88).—**1.** $\frac{2}{3}$; $\frac{3}{4}$; $\frac{1}{5}$. **2.** $\frac{9}{16}$; $\frac{3}{5}$; 9. **3.** $\frac{1}{7}$; 7; $\frac{1}{4}$. **4.** $\frac{4}{5}$; $\frac{9}{20}$; $\frac{5}{8}$. **5.** 2; $\frac{1}{3}$; $\frac{33}{70}$. **6.** 64. **7.** $4\frac{5}{6}$. **8.** 3 : 10; 27 : 2; 1 : 4; 3 : 10; 11 : 35.

Examination Paper No. XIII.—(Page 88) —**1.** $11.0\dot{6}65483\dot{}$. **2.** $3.\dot{6}$. **3.** \$505.30 (term of discount being $2\frac{1}{10}$ mo.) **4.** 30 sq. rd. 2 sq. ft. 72 sq. in. **5.** $52\frac{3}{5}$c.

Examination Paper No. XIV.—(Page 88) —**1.** \$6000, \$5000, \$4000, respectively. **2.** \$104.3125 loss. **3.** $3\frac{97}{101}\%$. **4.** $1.22\dot{6}$. **5.** 317.

Exercise XXXVII.—(Page 90).—**1.** 16; 40. **2.** 6; $6\frac{3}{4}$. **3.** 36; 15. **4.** $38\frac{2}{3}$, $1\frac{19}{21}$. **5.** 27 yd.; $13\frac{1}{8}$ lb. **6.** \$10.50. **7.** £1 2s. **8.** $\frac{1}{264}$.

Exercise XXXVIII.—(Page 91).—**1.** 25 mi. **2.** 166.2+ yd. **3.** 32A. **4.** $24\frac{3}{5}\frac{4}{5}$c. **5.** 4800. **6.** \$3.50. **7.** \$192.855. **8.** \$10. **9.** \$7. **10.** \$2.46. **11.** \$62.72. **12.** 4.

Exercise XXXIX.—(Page 93).—**1.** 58. **2.** 15. **3.** 23. **4.** $6\frac{1}{4}$. **5.** 46. **6.** $\frac{7}{8}$. **7.** 17. **8.** .3.

Exercise XL.—(Page 94).—**1.** 165. **2.** 324. **3.** 168. **4.** 1550. **5.** $237\frac{1}{2}$. **6.** 5050. **7.** 6400 ft.; 624 ft. **8.** \$409.50.

Exercise XLI.—(Page 96).—**1.** 256; 512. **2.** 16384; 262144. **3.** 256; 4096. **4.** $3\frac{1}{8}$; $\frac{25}{256}$. **5.** $1\frac{47}{81}$; $2\frac{539}{729}$; $4\frac{6524}{6561}$. **6.** $\frac{1}{16384}$. **7.** $\frac{1}{2}$. **8.** $341\frac{1}{3}$.

Exercise XLII.—(Page 97).—**3.** 349525. **4.** $1\frac{243}{512}$. **5.** $238\frac{1}{8}$. **6.** 2047. **7.** $39\frac{123}{128}$. **8.** $2\frac{1803}{4096}$. **9.** \$1.27. **10.** \$31000. **11.** \$1023.

Exercise XLIII.—(Page 98).—**1.** 24. **2.** 1. **3.** 18. **4.** 625. **5.** 1024. **6.** $7\frac{1}{2}$. **7.** $111\frac{1}{9}$. **8.** $1\frac{1}{9}$. **9.** $\frac{5}{9}$. **10.** $\frac{4}{11}$.

Exercise LXIV.—(Page 99).—**1.** $524.05; $84.05. **2.** $1125.508; $125.508. **3.** $760.725; $70.725. **4.** $1012.255; $212.255. **5.** $2649.79; $249.79. **6.** $1854.405; $54.405.

Exercise XLIVa.—(Page 101).—**1.** $2153.85; $2800. **2.** $3240; $2700. **3.** $1567.50; $1381.06. **4.** $7520; $5371.42+. **5.** $292.886. **6.** $5607; $4441.188+. **7.** $408. **8.** $500. **9.** $136. **10.** $1284.

Exercise XLV.—(Page 103).—**1.** $1261; $1089.30. **2.** $1312.38; $1039.53. **3.** $2809. **4.** $1406.08. **5.** $500.

Exercise XLVI.—(Page 104).—**1.** 470. **2.** 120 ft. **3.** 160 rods. **4.** 75 yd. **5.** 36. **6.** 56.64 ft. **7.** 52.2 ft. **8.** .4099^{Hm}. **9.** 1.83^{Dm}. **10.** 494.18 + mi.

Exercise XLVII.—(Page 105).—**1.** 3.6A. **2.** $10.97. **3.** $5.40. **4.** 276$\frac{4}{7}$ rods. **5.** $120.866. **6.** $2750. **7.** 50c. **8.** 7.755 yd. **9.** 17 suits and $\frac{1}{2}$ yd. over. **10.** $17.96. **11.** $83.73. **12.** $556.76. **13.** $506.88. **14.** $161190. **15.** 500 sq. ft. **16.** 6.01634. **17.** 8$\frac{1}{2}$ sq. ft. **18.** 60 yd. **19.** 15 yd. **20.** 1000 sq. ft.

Exercise XLVIII.—(Page 107).—**1.** 2352.5 + sq. ft. **2.** 15327.98 sq. yd. **3.** 111.8 sq. ft. **4.** 13.26 A. **5.** 10 A. 73.12 + rods.

Exercise XLIX.—(Page 107).—**1.** 81.6816 in. **2.** 31.416 ft. **3.** 24 ft. 4.168 in. **4.** 5000 ft. **5.** 2827.44 sq. ft. **6.** 962.115 sq. ft. **7.** 1A. 94.46 rd. **8.** 3183.1 sq. ft. **9.** 19 A. 143 + rd. **10.** 75.006 sq. rd. **11.** 490.089 sq. ft. **12.** $157.08.

Exercise L.—(Page 108).—**1.** 81. **2.** 168$\frac{3}{4}$. **3.** 10000. **4.** 66$\frac{2}{3}$. **5.** 1020. **6.** 249$\frac{9}{16}$.

Exercise LI.—(Page 109).—**1.** 34.328 cu. in.; 63.375 sq. in. **2.** 1360 lb. **3.** 722.2 + lb. **4.** 16.478^{Kg}. **5.** $182. **6.** 247.27\frac{1}{2}$. **7.** $27.69. **8.** 1944. **9.** 74.78 + bus. **10.** 14. **11.** 3 ft. 8 + in. **12.** 1 T. 8 cwt. 4.4 lb.; $46.74.

Exercise LII.—(Page 110).—**1.** 197.9208 cu. ft. **2.** 8 ft. **3.** 549.78 cu. ft. **4.** 8.83575 cu. ft. **5.** $29.23. **6.** 11 ft. **7.** 305.4672 cu. m. **8.** 8181.25 cu. in. **9.** 1272.348^{g}. **10.** 233.98 + lb. **11.** 122.786 + liters. **12.** Conical part, 65.45 cu. in.; frustum, 458.15 cu. in.

Exercise LIII.—(Page 111).—**1.** 2.4. **2.** 2.73. **3.** 4. **4.** 5. **5.** 1468.8^{Kg}. **6.** 316.4^{Kg}. **7.** .64. **8.** 1.398. **9.** 3.172. **10.** $\frac{5}{12}$. **11.** .6. **13.** 2.1625. **14.** .844.

EXAMINATION PAPERS.

15. (Page 112).—1. $3.28+. 2. .0$\dot{4}$. 3. 1s. 7d. 4. 24 yd. 5. .822.

16. (Page 112).—1. $\frac{11}{330}$. 2. $\frac{4}{17}$, $15000. 3. $4000. 4. £32 14s. $7\frac{3}{4}$d. 5. 800.

17. (Page 113).—1. $1215. 2. $6.81. 3. 1561.065. 4. 49.9125. 5. 11 ft. $8\frac{1}{2}$ in.

18. (Page 113).—1. 96. 2. 8.686875. 3. $394.35. 4. 224.6°F. 5. −35C.

19. (Page 113).—1. $3\frac{6}{47}$ da. 2. $304.76. 3. $437.58. 4. $327.82. 5. 3 yr. 46 days.

20. (Page 114). 1. $32\frac{8}{11}$ min. past 12. 2. A. $1000. B. $1200, C. $1400. 3. A. $77.42, B. $129.03. C. $193.55. 4. $199.66, $2.44. 5. 5%.

21. (Page 114).—1. 713. 2. 160. 3. $\frac{35}{48}$. 4. $\frac{133}{190}\frac{33}{80}$. 5. $16.21.

22. (Page 115).—1. $13\frac{1}{2}$. 2. $\frac{27}{64}$. 3. $\frac{3}{4}$. 4. $23\frac{7}{16}$ 5. Take 100 feet as the depth of a lot and measured on the breadth of the field. The latter is more advantageous by $1168. Fraction left over is $\frac{38}{303}$.

23. (Page 115). 1. $21\frac{3}{5}$d. 2. $59.40. 3. $13\frac{1}{3}$ cents. 4. $40,000, $36,000, 4,000, 3,600 respectively. 5. $2364.

24. (Page 116).—1. $1.76. 2. 210700; 1.9992194. 3. 1 ft. 4 in. 4. £615. 5. $600; $13\frac{1}{2}$ yr.

25. (Page 116).—1. 13 min. past 11. 2. 16000. 3. $1.60. 4. 8100. 5. $3631.65.

26. (Page 117).—1. $4.01. 2. 70. 3. The former by $.04+. 4. 15. 5. $.437.

27. (Page 117).—1. 12%. 2. 32146 sq. yd. 3. 68800. 4. $56.47. 5. 1 ft. 9.8 in.

28. (Page 117).—1. 1 hr.; 3, 4, and $4\frac{1}{2}$ miles respectively. 2. $9\frac{1}{11}$%. 3. $36720. 4. $55.80. 5. $653\frac{1}{3}$.

29. (Page 118).—1. 113.0976. 2. $7.50. 3. $34\frac{1}{6}$c. 4. $1333\frac{1}{3}$.

30. (Page 118).—1. 6560. 2. 14196. 3. 50%. 4. 4%. 5. 5%.

31. (Page 119).—1. (A) In 5 hr.; 125 mi. from Halifax.
(B) In $4\frac{3}{11}$ hr.; also in $5\frac{8}{11}$ hr.
(C) $88\frac{7}{11}$ mi. from St. John.
2. 14457$\frac{1}{7}$ sq. ft. 3. 9%. 4. $35.10. 5. 224.6°F., 352.4°F.

32. (Page 119).—1. 34$\frac{3}{7}$ min. past 13 on Wednesday; 37° 51$\frac{3}{7}$′N. 3. 7202.9. 4. 949.47. 5. 5$\frac{1}{2}$ cents (nearly).

33. (Page 120).—1. 12 $\frac{3}{4}\frac{3}{1}\frac{1}{1}$. 2. 3115. 6.885262048727389. 3. 7833\frac{1}{3}$. 4. 18 da. 5. $867.92.

34. (Page 120).—1. 60%. 2. 6 mo. 3. $165.71. 4. 348.8°F. 5. 280.

35. (Page 121).—1. 52 yr. 2. $1381. 3. £19 16s. 6.912d. 4. 304. 5. 299$\frac{1}{4}$.

36. (Page 121).—1. 15 da. 2. $783, 3$\frac{1}{3}$ yr. 3. Ans. £15 2s. 2$\frac{3}{5}$d. 4. 1296.56\frac{1}{3}$. 5. $.0579.

37. (Page 122).—1. 40 mos. 2. $132.23. 3. 201$\frac{17}{153}$. 4. $4500; $33.75. 5. 8724.9.

38. (Page 122).—1. 1.7099. 2. 19$\frac{1}{3}$. 3. $203.105. 4. 1$\frac{1}{16}$%. 5. 1st, .26795 ft.; 2nd, .31784 ft.; 3rd, .41421 ft.

39. (Page 122).—1. 35°C., 350°C. 2. $99.00. 3. $30300. 4. 49° 41′ W. 5. .8099.

40. (Page 123).—1. 30375. 2. 61.236. 3. 1142\frac{4}{7}$. 4. 8.33\frac{1}{3}$. 5. 517 lb.

41. (Page 123).—1. 63.8 rods. 2. $158. 3. The 90th meridian west, the 90th east and the 180th respectively. 4. 70$\frac{5}{24}$ sq. ft.; 29$\frac{1}{8}$ cu. ft. (The box has a lid). 5. 5$\frac{15}{17}$ ft.

42. (Page 124).—1. A. 183\frac{1}{3}$, B. 33\frac{1}{3}$, C. 33\frac{1}{3}$. 2. $\frac{37}{60}\frac{8}{5}$. 3. 12$\frac{1}{2}$%. 4. 11109. 5. 45.4+ liters.

43. (Page 124).—1. 140 yd. 2. 85. 3. 1134 sq. ft. 4. $2500. 5. £553$\frac{11}{13}$, £369$\frac{3}{13}$, £276$\frac{12}{13}$ respectively.

44. (Page 125).—1. 58 min. 48 sec. past 10 o'clock, Dec. 31,'97. 2. June 6th. 3. $511. 4. 60606\frac{2}{33}$. 5. 22$\frac{2}{23}$ min. past 4.

45. (Page 125).—1. 6$\frac{2}{3}$%. 2. A. $437, B. $414. 3. 150. 4. 152. 5. 4.6, nearly.

46. (Page 126).—1. $539.45. 2. $1620. 3. $653.58; $756.60. 4. $\frac{3}{5}$ gallon. 5. 65.

47. (Page 126).—1. 80. 2. 7$\frac{1}{32}$%. 3. The former. 4. 4%. 5. .0464.

48. (Page 127).—1. $3.20. 2. 11 miles. 3. 121 yd.; 242 yd. 4. 81$\frac{1}{4}$ cents. 5. 27$\frac{1}{13}$ ft.; 1$\frac{7}{33}$ sec.

49. (Page 127).—1. 83.776 sq. ft. 2. 20 mi.; 2$\frac{1}{2}$ mi. per hour. 3. A. $896, B. $1512, C. $2940. 4. 17.32 ft., 24.49 ft. 5. Sept. 25.

50. (Page 128).—**1.** $7\frac{77}{405}$. **2.** $50. **3.** A. $1714\frac{2}{7}$, B. $2285\frac{1}{7}$. **4.** $11\frac{1}{7}\%$. **5.** $4480.

51. (Page 129).—**1.** 15 da. **2.** $1.35. **3.** 144. **4.** $\frac{5}{8}$. **5.** 492 yd.

52. (Page 129).—**1.** 2, 4, and 8 feet. **2.** $1.73\frac{1}{3}$, $.93\frac{1}{3}$, $.33\frac{1}{3}$. **3.** $10\frac{10}{23}$ and $41\frac{17}{23}$ min. past 22. **4.** $53\frac{1}{3}$ ft. **5.** $2791.19.

53. (Page 130).—**1.** 1 hr. 40 min. after it starts. **2.** $19\frac{1}{4}\frac{3}{3}$. **3.** $37.80. **4.** 315 lb. nearly. **5.** 38.27.

54. (Page 130).—**1.** A. $1822.50; B. $1701; C. $1417.50. **2.** $55.65. **3.** A. $609.375. B. $693.75, C. $759.375. **4.** $356.99. **5.** 4.

55. (Page 131).— $12\frac{1}{2}\%$. **2.** $2047.50. **3.** $1520. **4.** $441.44. **5.** 307200.

56. (Page 131).—**1.** $32\frac{8}{807}$. **2.** A. $17.50, B. $12.50. **3.** $10,000 and $20,000 respectively. **4.** 3 yr. 2 mo. **5.** 48 inches.

57. (Page 132).—**1.** $\frac{95}{105}$ of value of flour. **2.** 4095. **3.** 720. **4.** 1767.15 cu. in. **5.** $1\frac{1}{8}$ da.

58. (Page 132).—**1.** 1168521. **2.** 20 ft. **3.** Feb. 1. **4.** $21\frac{7}{8}\%$. **5.** The latter $60 more.

59. (Page 133).—**1.** $36 **2.** 34512. **3.** 456. **4.** £11 16s. $4\frac{4}{11}$d. **5.** 10 min. past 12; $86\frac{2}{3}$ miles from X.